MAD 2104 Oct 11, 2012
Quiz 4 and Key Prof. S. Hudson

1) [25pts] Give a recursive definition for the sequence $1.5, 2, 3, 6, 18, 108, \ldots$

2) [20pts] Suppose our class consists of 24 teenagers and 34 Dexter fans, including 16 teenaged Dexter fans. How many people are in our class ?

3) [25pts] Compute the Boolean product $A \odot B$, showing work, where

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

4) [30pt] Choose ONE proof. Use induction. You may answer on the back.

a) For every positive integer $n$, $1 + 3 + 5 + \cdots + 2n + 1 = (n + 1)^2$.

b) For every positive integer $n$, 3 divides $n^3 - n$.

c) Any defective $2^n$ by $2^n$ chessboard, with $n \geq 1$, can be tiled by triominoes.

**Remarks and Answers:** The average grade was approx 88 out of 100, based on the top 25 scores. The top two scores were 100, 100. This is very high for Quiz 4.

A's 92 to 100
B's 84 to 91
C's 74 to 83
D's 64 to 73

Again, I have estimated your semester grade in the upper right corner, based on your quizzes so far. I changed systems this time by dropping your lowest grade. That, plus the high scores on this quiz, produced a very high average (approx 84) and a high scale. The scale usually comes down to match the one on the syllabus by the end of the term, and the estimates also gradually become more accurate.

1) *Basis)* Let $a_1 = 1.5$ and $a_2 = 2$.
*Rec)* For $n \geq 3$, let $a_n = a_{n-1}a_{n-2}$.

Remark: Many people didn't see the pattern. After that, the most common mistakes were to omit the Basis step (fairly serious) or the phrase about $n \geq 3$ (not so serious). Several variations of my answer are OK, but some choices of notation aren't so good, such as $p(n)$ or $a_x$.

2) $34 + 24 - 16 = 42$. Almost everyone got this.

3) Most answers were correct, but a few people confused $\wedge$ and $\vee$.

$$A \odot B = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

4a) See the book or lectures for similar proofs. A common notational mistake (that I warned against) was to write $p(1) = \ldots$ (followed by a number or whatever. It is already wrong). It is OK to write $p(1)$ says ..., or $p(1)$ is ... (some proposition). This was -4 points. Another common mistake (that I warned against) was not including enough words. For example, your proof should contain 'Induction Step' followed soon by 'Assume' (etc). I will not tell you exactly which words to use, but you should make approx the same verbal effort as in the textbook and lecture examples. Similar remarks apply to 4b.