MAD 2104 | July 19, 2012
Quiz 4 and Key | Prof. S. Hudson

1) [20 pts] Give an example of two uncountable sets A and B such that $A \cap B$ is finite.

2) [20 pts] Give an example of a set A and a function $f : A \to A$, that is one-to-one but not onto.

3) [30 pts] True-False

The set of all finite bit strings is countable.

If $S \cup T \subset$ the domain of f, then $f(S \cap T) = f(S) \cap f(T)$.

If $S \cup T \subset$ the domain of f, then $f(S \cup T) = f(S) \cup f(T)$.

For all sets, $A \bigoplus B = (A \setminus B) \cup (B \setminus A)$.

If $f : A \to B$ is onto, then $f(A) = B$.

4) [30 pts] Choose ONE induction proof (if you answer on the back, leave a note here):

a) Conjecture a formula for the sum of the first n positive odd integers. Then prove your conjecture by induction.

b) Prove that $1^2 + 3^2 + \ldots + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$ for every nonnegative integer n.

Bonus (approx 5 pts): How many positive integers less than 1000 are divisible by either 7 or 11 ?

Remarks and Answers: The average was about the same as for Quizzes 2 and 3. The two highest grades were 103 and 105 ! The unofficial scale for the quiz is

A's 80-100
B's 70-79
C's 60-69
D's 50-59

I have estimated your semester grade, based on your best 3 out of 4 quiz scores, and noted it on the upper right corner of your Quiz. This method assumes you have already had your lowest grade, which may not be correct. Also, I have not yet included your HW grades. If you have been excused for missing a quiz, or have had a grade changed, this estimate may be inaccurate. However, for most people, it is probably fairly accurate, and will be pretty close to your final semester grade, unless, for example, you change your study habits a lot.

Since the new calculation method raises the average, the A's start at 90, for this estimate. That number will probably come down rapidly as more quiz scores are included. Note - this estimate is just advisory, not really an official grade of any kind. Feel free to see me about this during office hours.

1) Let A and B be two disjoint intervals such as $[0, 1]$ and $[3, 4]$, so that $A \cap B = \emptyset$ which is finite. There are lots of other examples. I could not give much partial credit for wrong answers on Problems 1 or 2 (or 3). See Ch.2.5-11.

2) Let $A = \mathbf{Z}^+$ and $f(n) = n^2$ which is one-to-one (no negatives in A) and not onto, since $f(n)$ cannot equal 2. You could also use $f(n) = n + 1$ or $f(n) = 2n$, etc. You could use some other set, but it has to be infinite, to find an f that works.

3) TFTTT. See Ch.2.3-40.

4) See Ch.5.1-5, and Ex 2. Most of the proofs were OK, but some common mistakes were a) not enough words, and b) the algebra, and c) bad notation, such as $p(0) = 1$. If you want help with this kind of problem, you are welcome to see me or Andres. Consider bringing in some of your work us to review.

Bonus) The number divisible by 7 is $\lfloor 999/7 \rfloor = 142$. If you haven't seen this, you might be able to reason this out by playing with simpler examples (how many numbers less than 20 are divisible by 7?). Likewise, there are $\lfloor 999/11 \rfloor = 90$ divisible by 11. The ones divisible by both are the ones divisible by 77, and there are $\lfloor 999/77 \rfloor = 12$ of those. So, the answer is 142+90-12= 220. See Ch.6.1-22.