

1a) [20pts each] Suppose k integers are chosen from the first eleven positive integers. Show that if $k \geq 8$, then some pair of the chosen integers has a sum equal to 12.

1b) Find the smallest value of k that makes the conclusion true. Justify your answer briefly.

2a) [15pts each] A coin is tossed 10 times, where each toss comes up heads or tails. How many possible outcomes are there?

2b) How many are there, with at most 2 heads ?

3a) [15pts each] Let $R = \{(1, 2), (3, 1), (2, 1), (1, 3), (2, 2)\}$ be a relation on $A = \{1, 2, 3\}$. Is R symmetric ?

3b) Is it transitive ? (justify briefly)

Remarks and Answers: The average among the top 20 students was about 65 / 100. Here is a rough scale for the quiz, based mainly on that average:

As 75 to 100

Bs 65 to 74

Cs 55 to 64

Ds 45 to 54

1a) The pigeons are the k chosen integers. The pigeonholes are the 6 sets $\{1, 11\}$, $\{2, 10\}$, $\{3, 9\}$, $\{4, 8\}$, $\{5, 7\}$ and $\{6\}$. By the PHP (since $k > 6$), some set contains two of the chosen numbers (and it is obviously not the last set), and those sum to 12.

1b) This also works if $k = 7$ but not if $k = 6$ (then the chosen integers could be the first 6). Problem 1 is similar to HW problems 5.2.14-16.

2a) $2^{10} = 1024$

2b) $C(10, 0) + C(10, 1) + C(10, 2) = 1 + 10 + 45 = 56$. I wanted you to simplify 2b), since it isn't hard, and is part of Discrete Math. I did not insist on simplification in 1a), since that may be a bit tedious. You can always ask about this kind of thing. Problem 2 is similar to HW problem 5.3.19.

3a) It is symmetric (no justification was required).

3b) It is not transitive. You should find a counterexample, such as: $1R2$ and $2R1$, but not $1R1$. Problem 3 is similar to HW problem 8.1.3.

NOTE: A few students mis-interpreted problems on Quizzes 4 and 5 (though none made the same mistake, as far as I know). You should be able to avoid this, by doing the HW and checking your answers. Most of these counting problems are asked in fairly standard and similar ways, and you can get used to that with practice. Also, you can ask me about interpretations during the quiz.