MAD 2104 Oct 27, 2011
Quiz 5 Key Prof. S. Hudson

1) [30pts] How many ways can 8 men and 5 women be placed in a line, such that no two women stand next to each other ? Hint: place the men first.

2) [20pts] In class, we proved a theorem about monotone subsequences. Using the notation of that proof, find (d_3, i_3) for the sequence 3, 6, 9, 4, 7, 10, 5, 8, 11, 12.

3) [20pts] How many ways can you put 5 eggs into 3 baskets, A, B and C ?

4) [30pt] Choose ONE proof.

a) Determine which amounts of postage can be formed from 3 cent and 5 cent stamps. Prove your answer using Strong Induction.

b) Give a combinatorial proof [Hint: count committees-with-a-leader two ways] that for all positive integers n,

$$\sum_{k=1}^{n} kC(n,k) = n2^{n-1}$$

Remarks and Answers: The average of the top 20 grades was approx 50, and the highest was 70, which suggests this was a relatively hard quiz. The lowest scores were on Problem 1, which (I thought) was a fairly standard counting problem. The unofficial scale is;

A's 65 - 100
B's 55 - 64
C's 45 - 54
D's 35 - 44

The number in the upper right is the sum of your best 4 out of 5 quiz grades. The average result for this number is approx 290 out of 400. I estimated your semester average from that, based on;

A's 330 - 400
B's 290 - 329
C's 250 - 289
D's 210 - 249

You may notice that the scale is now fairly close to the scale on the syllabus (a bit lower). Based on past experience, Quiz 6 and the final will probably bring it down a bit more, but the HW scores may lift the scale a little.

1) $8! \cdot P(9,5) = 8! \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$. This was 5.3.23 (not assigned). There are $P(8,8) = 8!$ ways to place the men in a line first (see Hint), which leads to the diagram below. It shows

there are 9 places to put Woman 1, then 8 places remain for Woman 2, etc.

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Most grades were very low, mostly zeros. It was difficult to give partial credit, since most answers were way off, without much explanation. I gave no credit unless;

* it was clear you were using the Product Rule (no additions or subtractions), and

* your answer was a multiple of 8! (and therefore, over 40,000 or so)

One fairly common answer (worth 10 points) was 8!5!. But this answers a different question, where the 5 women follow the 8 men.

2) $(3, 4)$. The longest decreasing subsequence starting from $a_3 = 9$ is 9,7,5, which has length 3. So, $d_3 = 3$, and likewise, i_3 is the length of 9, 10, 11, 12 which is 4. This uses the notation in the proof of Theorem 3 of Ch.5.2, on page 352, which we also did in class.

3) 21. We did this one in class; it is the same as counting the bit strings of length 7 with exactly 2 ones, so $C(7, 2) = 21$. I feared low scores on this (and Problem 2) but the average score on this one was good.

4a) This is similar to various HW exercises and to Ch 4.2. Ex4, which we did in class. See the *bottom* of page 287 for the Strong Induction proof, which is simpler than the regular induction proof higher up on the page.

The possible postages are 3, 5, 6, 8, 9, 10, etc. These (especially 8, 9 and 10) can form the Basis step. The Ind step involves adding 3 cents to some previous case, and it can start at $k = 11$ (but you should explain this in much more detail, as done in class, or on page 287). Some common mistakes were;

Not listing all the postages (omitting 3, 5 and 6, for example)

Using regular induction instead of strong induction (see pg 287 to contrast the 2 methods).

Not following the standard induction format (failure to state the IH clearly, for example)

Bad notation, such as $P(8) = 5 + 3$.

4b) This is HW 5.4.29, but few chose it. LHS = ways to choose a committee (of k members) followed by choosing the leader. RHS = ways to choose the leader, followed by the other members (some additional explanation left to you).