MAD 2104 Oct 25, 2012
Quiz 5 Prof. S. Hudson

1) [25pts] How many numbers must be chosen from the list $\{3, 4, 5, 6, 7, 8, 9, 10\}$ to be sure at least two add up to 11? Prove your answer using the PHP or GPHP (to be totally clear, describe exactly what the pigeons and pigeonholes are).

2) [45pts] How many bit strings of length 6

a) Have the same number of 0s and 1s ?

b) Have an odd number of 1s ?

c) Contain two consecutive 1s ? [Hint: the number that *don't* is a Fibonacci number.]

3) [30pts] Answer True or False. Recall that $r(n, m)$ denotes a Ramsey number.

$r(4, 2) = 4$

$r(3, 3) = 5$

For all integers with $1 < k < n$, $P(n, k) = P(n-1, k) + P(n-1, k-1)$

$\sum_{k=0}^{10} 2^k C(10, k) = 3^{10}$

Some relations are both symmetric and anti-symmetric.

Remarks and Answers: The average grade was approx 60 out of 100, based on the top 25 scores. The top two scores were 94 and 89. This is fairly normal, but a bit low, for Quiz 5.

A's 75 to 100
B's 65 to 74
C's 55 to 64
D's 45 to 54

Again, I have estimated your semester grade in the upper right corner, based on your best 4 out of 5 quizzes so far. The average result was about 78.5. The best two results were both 96. I used the scale below, which has dropped 6 points from last time, but is still a few points above the the one on the syllabus. If your HW grades don't match your quiz scores, eg much above or below, you should adjust my estimate for that.

A's 88 to 100
B's 78 to 87
C's 68 to 77
D's 58 to 67

1) At least 6 (these are the pigeons). The pigeonholes are the 5 sets $\{3, 8\}$, $\{4, 7\}$, $\{5, 6\}$, $\{9\}$, $\{10\}$. Since $6 > 5$, two of the chosen numbers belong to the same set (therefore one of the first three sets), so they add up to 11.

It is not very hard to find 6 by trial and error (eg, pick 3, 4, 5, 9, 10, so far avoiding any sums that make 11, but then you are stuck at 5). I gave approx half credit for that. But for full credit, you had to list 5 pigeonholes, and explain using the PHP. A few people seemed confused by the wording of the problem, but it is similar to ones in the HW and exercises. This is one good reason to *scan* all the exercises, even if you don't have time to do them all.

2a) $C(6,3) = 20$.

2b) $C(6,1)+C(6,3)+C(6,5) = 6+20+6 = 32$. You can also say 'half of them', $2^6/2 = 32$, but then you should probably explain why this method works.

2c) 43. We saw in class that 13 bit strings of length 5 do not contain '11', and that these answers are Fib numbers. So, the length 6 answer is 21. Then 64-21=43.

3) TFFTT