MAD 2104 July 26, 2012
Quiz 5 and Key Prof. S. Hudson

1 [20pts each] a) Give a recursive definition of the set of positive integers that are divisible by 7.

1b) Give a recursive definition of the sequence $a_n = 2 + 3(-1)^n$ for $n = 1, 2, 3 \ldots$.

2) [15pts each] a) How many ways can we choose a committee of k people from a group of n people, and then a leader from among those k people ? (explain)

2b) How many ways can we choose a leader from among n people, and then a committee of $k - 1$ other people (to complete a committee of k people similar to 2a) ? (explain)

3) [15pts each] a) Use the matrix M_R to draw a graph that represents the relation R on the set $A = \{1, 2, 3, 4\}$.

$$M_R = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

3b) Decide whether R is transitive and justify your answer.

Remarks and Answers: The average among the top 26 scores was 63 out of 100, fairly similar to Quiz 4. The high grade was 90. The scale for this quiz is

A's 75 to 100
B's 65 to 74
C's 55 to 64
D's 45 to 54

The average for the semester is 72 out of 100 (based on each student's best 4 of 5 quiz scores - with the top 26 scores included). One student has an average over 100. I wrote your average and letter grade in the upper right corner. The scale for that is about 10 points higher than the one for Quiz 5 above (and it now matches the syllabus pretty well).

1a) Let $S \subset Z^+$ such that (B) $7 \in S$ and (R) if $x, y \in S$ then $x + y \in S$. [It would be OK to write $x + 7 \in S$ instead]. A very common mistake was to define a *sequence* instead of a *set*, for example with $a_n = a_{n-1} + 7$, etc. See 5.3-23.

1b) (B) Let $a_1 = -1$ and $a_2 = 5$. (R) For $n > 2$, let $a_n = a_{n-2}$. There are several other good answers, such as

(B) Let $a_1 = -1$. (R) For $n > 1$, let $a_n = 4 - a_{n-1}$. There was a version B of this quiz, with a similar 1b, just with different numbers. See also 5.3-8b.

2a) Two decisions, with $C(n,k)$ options, followed by k options, so $kC(n,k) = n!/(n-k)!(k-1)!$ ways. A common mistake was to answer for the two decisions separately, but not combine them. You did not have to simplify much, but I took a point off for $C(k,1)$ instead of k.

2b) There are n options, followed by $C(n-1,k-1)$ options, so $nC(n-1,k-1) = n!/(n-k)!(k-1)!$ ways. Since 2a) and 2b) are really the same question (perhaps slightly in disguise), they have the same answer after simplification. I intended to accept 'same as 2a' as a valid answer to 2b, but nobody said that.

These answers give a *combinatorial* proof that $kC(n,k) = nC(n-1,k-1)$, see Ch 6.4-21. The *algebraic* proof is the pair of simplifications above.

3) There were two versions of the quiz, with two different matrices. Either way, most people got the graph right. It should have 4 vertices, labeled 1 to 4, with 7 arrows.

It is not transitive. For this version of the quiz, you could remark that $1R4$ and $4R1$ but not $1R1$ (and there are other counterexamples).