

1) [30pts] Find the number of edges in each of these special graphs. Show work (hopefully not just a drawing) and label your answers clearly.

a)  $G = Q_5$

b)  $G = K_7$

c)  $G = W_9$

2) [40pts] Define  $R = \{(a, b) \in N \times N : a|b\}$ , a relation on the set of natural numbers. Define  $(a, b) \in S$  to mean  $a < b$ .

a) Describe the relation  $R \circ S$  as simply as possible. [If you cannot describe it, list two ordered pairs that belong to it, and two that do not, with reasoning].

b) Circle the properties that  $R$  has (and only those), among these four. You do not have to justify your answers.

reflexive, symmetric, antisymmetric, transitive

3) [30pt] Prove Thm 1 from 8.1;  $R$  is transitive on  $A$  if and only if  $R^n \subseteq R$  for  $n = 1, 2, 3, \dots$ . You can answer on the back.

**Remarks and Answers;** The average was about 60 / 100. The scale is;

A's 70-100    B's 60-69    C's 50-59    D's 40-49

I've averaged your best 5 of 6 quiz grades in the corner. This does not yet include the HW or the final. The class average for this stat is approx 70 / 100. The scale for it is

A's 80-100    B's 70-79    C's 60-69    D's 50-59

You should check that your average is correct, and [check online] that you are registered for the course. Please see me if you do not understand anything about your grade. You can also check with me soon, or our LA now, about your HW grades. Note that any extra-credit projects are due in about a week.

If you want to 'plan for the final' remember that it counts 30 points, which is half as much as your quiz average. For example, if you have a 60 now, but plan to get a 90 on the final, then your average after that will be  $(60+60+90)/3 = 70$  (and your HW will have some effect too). I cannot predict what the scale will be then, but I do not expect it will change much from the scale above.

1) 80, 21, 18 [10 points each]. For full credit, use valid formulas to get these. Otherwise, draw pictures and ‘count with your fingers’ for partial credit. See 9.2 for the notation used here. See the lecture notes for the formulas or similar examples.

2a)  $R \circ S = S$  ; see me for an explanation. Partial credit (small) for stating the def of  $R \circ S$ ; more partial credit (up to 15 out of 20) for listing 4 pairs, with correct reasoning, eg  $(2, 10) \in R \circ S$ , because  $(2, 5) \in S$  and  $(5, 10) \in R$  etc. This is similar to HW in 8.1.

2b) It is ref, anti-s, trans, so it is a partial order, but it is not symm. Each of the 4 parts was 5 points.

3) See textbook. Some people got confused over the ‘if’ and the ‘only if’, which is not unusual. It is OK, at least with me, to use  $\Leftarrow$  and  $\Rightarrow$  instead. The important thing here is to ASSUME  $R$  is transitive, and PROVE  $R^n \subset R$  in one part of your answer (use induction for this), and do the opposite in the other part (use  $n = 2$  and definitions).