

1) [10pts each] You are given the matrix M_R for a relation R below. Label your answers clearly.

$$M_R = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

- a) Find the matrix for R^{-1}
 - b) Find the matrix for \overline{R}
 - c) Find the matrix for R^2
 - d) Decide whether R is transitive (a graph may help), and explain.
- 2) [30pts] Draw the graph $K_{4,4}$. Decide if it is bipartite and explain your answer.
- 3) [30pts] Choose ONE: (you can answer on the back):
- a) Prove by induction that if R is transitive on a set A , then $R^n \subseteq R$ for all positive integers n .
 - b) Suppose S is a finite set of n elements. Show that exactly half of its subsets have an odd number of elements.

Remarks: The average was 74, with 4 perfect scores. The scale for Quiz 6 is

A's 85 to 100
B's 73 to 84
C's 63 to 72
D's 53 to 62

The semester average, based on the best 5 out of 6 quizzes of the top 25 students, is approx 77. The top two are 97 and 96. I have used this to estimate your semester grade in the upper right corner, as usual. Please check this and let me know if you don't get the same, or don't understand it. The current scale is:

A's 87 to 100
B's 74 to 86
C's 64 to 73
D's 54 to 63

Answers:

1abc) These refer to the inverse relation (take the transpose), complement, and $R \circ R$ (Boolean product).

$$M_{R^{-1}} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad M_{\bar{R}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad M_{R^2} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

1d) Not transitive. Eg bRa and aRb but not bRb (there are several other such failures).

2) Draw 4 vertices on the left and 4 on the right and connect every left to every right (16 edges). It IS bipartite, because you can color the left ones blue (or call them V_1) and color the right ones red (V_2). You should either write this out, or perhaps just label the left ones "B", etc. If you said $K_{4,4}$ is "bipartite by definition", I'd probably accept that explanation, but I don't recall that anyone said that.

3) See the text for 3a, which was advertised, and was the more popular choice. See me for 3b which is a little easier if you didn't prepare for 3a [one idea is to remove one element from S and compare its subsets with the ones of S].

I was curious whether choosing 3a was a sign of better study habits, so I compared the grades of the two groups. On average, the students who chose 3a scored 80 on Quiz 6. The ones who didn't averaged 50 (partly because the average scores were somewhat lower on 3b than 3a). So, there was a pretty strong correlation between studying for the assigned proof and overall success.

The ones who chose 3a also scored about 5 points above the rest on their semester average, but I don't think this is very significant, since it can be explained by the differences on Quiz 6. So there was no indication that studying hard for Quiz 6 equated with studying hard for the other quizzes.