MAD 2104 Aug 2, 2012
Quiz 6 and Key Prof. S. Hudson

1) [20pts each] a) Let A be the set of bit strings of length 5. Let R be the relation on A that two bit strings have the same number of zeroes. It is an equivalence relation. Is it symmetric ? antisymmetric ? Explain both answers briefly.

b) How many strings are there in the equivalence class $[01010]_R$?

2) [30 pts] Show that either K_4 or K_5 has an Euler circuit by drawing the graph and the circuit. If you cannot draw the circuit clearly, you can provide the vertex sequence instead.

3) [30 pts] Choose ONE proof: if you use the back, leave a note here.

a) Prove that if R is transitive on a set A, then $\forall n \geq 1, R^n \subseteq R$. (you don't have to prove the other part of Thm.1).

b) Give a *combinatorial* proof that $nC(n-1, k-1) = kC(n, k)$.

Bonus) How many relations R on $A = \{a, b, c\}$ are both symmetric and antisymmetric ?

Remarks and Answers: The average was about 61, a few points below Quizzes 4 and 5. The highest grade was 100. The unofficial scale for the quiz is

A's 75-100
B's 65-74
C's 55-64
D's 45-54

I have estimated your semester grade, based on your best 5 out of 6 quiz scores, and noted it on the upper right corner of your Quiz. The average among the top 25 students for these is about 70 out of 100. The A's start at approx 83.

1a) It must be symmetric, because it is an equivalence relation (or explain via the definition). It is not anti-symmetric because, for example, 00111 R 11100 but 00111 $\neq$ 11100. Note that some relations, even some equivalence relations are both (see the Bonus).

1b) There are 10 strings with exactly 3 zeroes, because $C(5, 2) = 10$. Most of the answers were correct, but a few people didn't understand that the R in 2b was the R in 2a. I can't see any other interpretation, but gave a little partial credit for counting the strings in A instead, eg $2^5 = 32$. As usual, an answer like 10 alone, with no work, did not get full credit.

2) K_5 has an Euler circuit because all vertices have degree 4, which is even (but K_4 has none). Apparently, about half the class knew what an Euler circuit was, and solved this rather easily. See Ch.10.5.

3a) See the textbook. 3b) See Quiz 5. If you used formulas with $n!$, you probably gave an *algebraic* proof, not a combinatorial one.

Bonus) $2^3 = 8$. If you think none exist, consider, for example $R = \{(b,b),(c,c)\} \subseteq A \times A$, which has both properties. Also, $R = \emptyset$, etc, do. Antisymmetry is not an exact opposite of symmetry.