MAD 2104 Dec 6, 2012

Final Exam Prof. S. Hudson

1) Show that $\neg(p \leftrightarrow q)$ is logically equivalent to $p \leftrightarrow \neg q$. Any standard style is OK, but include at least a sentence or two.

2) Determine whether each of these proposed definitions is a valid recursive definition of a function f from the set of nonnegative integers to the set of integers. If it is well-defined, find an explicit formula for $f(n)$. If not, explain why not.

a) $f(0) = 0$, $f(n) = 2f(n-2)$ for $n \geq 1$.

b) $f(0) = 1$, $f(n) = f(n-1) - 1$ for $n \geq 1$.

3) Suppose a saleswoman has to visit eight different cities. She must begin her trip in a specified city (you can assume it is Miami) but she can visit the other seven cities in any order she likes. How many possible orders can the saleswoman use when visiting these cities ?

4) Let $A = \{1,\ 3,\ 6,\ 10,\ 15,\ 21\}$. Define an equivalence relation R on A by $R = \{(a, b) \in A \times A : 3|(a-b)\}$ (meaning $a \equiv b \pmod 3$). List the elements of the equivalence class [10].

5) [15 points] Answer True or False:

The set $\{^{-},\ \cdot\}$ is functionally complete.

The set of irrational numbers is countable.

If R is an equivalence relation, then $R^2 = R$.

There is a simple graph G with degree sequence 5,4,3,3,3,2,1.

In Boolean algebra, $x(y + x) = x$.

6) Give an example of a weakly-connected digraph G that is not strongly-connected, and explain the difference.

7a) [10pts] Let $F(x, y, z)$ be the Boolean function equal to 1 if and only if an odd number of $\{x, y, z\}$ are 1's. Express F as a sum of products (eg in DNF).

7b) [5pts] Can your answer to 7a) be simplified using a Karnaugh map ? Explain. You may not have to actually draw a map to do so.

8) Express $y * ((x + 3) \uparrow 2)$ in postfix form.

9) Choose ONE proof. For full credit use the normal paragraph style (possibly with formulas, such as $x \in A$ etc, mixed in).

a) Prove that a strictly decreasing function $f : R \to R$ must be one-to-one.

b) Suppose $R \subseteq S$ are two relations on a set A. Recall that $R^{-1} = \{(b,a) \in A \times A : (a,b) \in R\}$. Show that $R^{-1} \subseteq S^{-1}$.

Remarks and Answers: The average among the top 23 was approx 71, which is pretty good for a final exam. The lowest scores were on problems 6 and 9 (averaging approx 40% each). I do not set an explicit scale for the final exam, but will scale the semester average. That scale will probably be similar to the last one announced.

1) Make a truth table and compute the 4 values for both expressions. Point out that they are the same for both (for most people, they were FTTF but this depends on your table). I accepted other styles, such as a sequence of changes (like trig identity proofs), but those did not all succeed.

2a) Not valid. We cannot determine $f(1)$ because there is no definition of $f(-1)$ (I accepted many variations of this explanation, but some answers just didn't make sense).

2b) Valid. $f(n) = 1 - n$.

3) 7!

4) $[10] = \{1, 10\}$

5) TFTFT

6) See the text. Most people who drew a digraph (even randomly) produced an acceptable picture. Few could explain the difference precisely, by mentioning the underlying undirected graph, but admittedly we did not practice much with these concepts.

7a) $xyz + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}z$ b) This cannot be simplified by a Kmap because no two minterms are adjacent.

8) $yx3 + 2 \uparrow *$ Include a picture of the tree for full credit (always show work).

9) These were unadvertised exercises, but not very hard. See the text or me about answers. A common mistake in 9b) was to begin with "Assume $(a,b) \in R$". It is better to start with "Assume $(b,a) \in R^{-1}$". Since these are equivalent, I didn't take off a lot of points, but the first version muddies the logic.