MAD 2104 Aug 9, 2012
Final Exam and Key Prof. S. Hudson

1) [10 pts] Find two numbers $m \geq 2$ and $n \geq 3$ so that Q_m is isomorphic to C_n. Draw the graph(s).

2) [10 pts] Recall that CAE is a *permutation* of ACE. How many permutations of the letters $ABCDEFGH$ contain the string ABC ?

3) [15 pts] Suppose $A = \{a, b, c, d, e\}$ and R is an equivalence relation on A with only two equivalence classes. Suppose $[b] = \{b, d, e\}$ is one of them. a) Compute $|R|$.

3b) Is R^2 an equivalence relation ? Explain briefly.

3c) Is $\bar{R}$ an equivalence relation ? Explain briefly.

4) [10 pts] Let n be an integer such that $3n + 2$ is odd. Prove that n is odd. Mention the type of proof you are using (induction ? contradiction ? etc).

5) [10 pts] a) Write out the $\sum$ formula for $(x + y)^{12}$ from the Binomial theorem. Choose x and y carefully (experiment with $x, y = 0$ or ± 1, etc) to calculate $\sum_{k=0}^{12} (-1)^k C(12, k)$.

b) Use part (a) to help calculate $\sum_{k=0}^{10} (-1)^k C(12, k)$.

6) [15 pts] True-False

The graph $K_{10,11}$ is bipartite.

The graph $K_{10,11}$ is planar.

We can form any postage of $n \geq 34$ cents from 7 cent and 9 cent stamps.

We can form any postage of $n \geq 8$ cents from 3 cent and 5 cent stamps.

89 is a term in the Fibonacci sequence.

7) [10 pts] Suppose a full 3-ary rooted tree T has 100 internal vertices. How many leaves does it have ?

8) [10 pts] Simplify this expression using a K-map, $w\bar{x}yz + \bar{w}x\bar{y}\bar{z} + w\bar{x}\bar{y}z + w\bar{x}\bar{y}\bar{z} + wx\bar{y}\bar{z} + \bar{w}\bar{x}\bar{y}\bar{z} + w\bar{x}y\bar{z} + \bar{w}xyz$.

9) [10 pts] Show that in a group of 10 people there are either 3 mutual enemies or 4 mutual friends. Hint: person A either has 6 friends or 4 enemies (why? how does this remark help? This was a HW problem, but see me if you don't understand it).

Bonus) [approx 5 pts] Choose ONE:

a) State and prove (in at least as much detail as done in class) Euler's Thm about existence of Euler circuits in a multi-graph.

b) Simplify the Boolean expression in problem 8 using the Quine-McCluskey method instead of a K-map. Include all the usual tables, show all your work, and comment briefly on your reasoning.

Remarks and Answers: The results were low on problems 5 and 9, but OK on most others. The highest grade was 90 and the average was approx 62. I do not need a separate scale for the final, but if so, would probably start the A's at approx 75. This exam was designed for 75 minutes, though we went about 7 minutes over that.

1) Q_2 is isomorphic to C_4. Both are squares (roughly speaking).

2) 6! See HW Ch 6.3.21, etc.

3a) 13. 3b) Yes, because $R^2 = R$ in this case. 3c) No, not reflexive.

4) In my opinion, the simplest method is to prove the contrapositive. See page 83. [Assume n is even, so $n = 2k$. Then $3n + 2 = 2(3k + 1)$ is even, too. Done].

The contradiction method is very similar, but probably needs more explanation, such as a clear assumption at the start that $3n + 2$ is odd. See page 87. Other methods might be possible, but less natural, like using a hammer to chop down a tree. I didn't give much credit for (unsuccessful) attempts to use induction or a trivial proof, etc, but one person was fairly successful using cases.

5a) $(x + y)^{12} = \sum_{k=0}^{12} C(12, k)x^{12-k}y^k$. Setting, $x = 1$ and $y = -1$, the second part of 5a is 0. This is Cor 2 on page 417. Note that the author makes a tiny mistake by using the exponent $n - k$ on y instead of x, but this really doesn't matter since x and y are practically interchangeable.

5b) 11. This sum + 2 more terms = 5a = 0. The last two terms are -12+1.

6) TFFTT. $K_{10,11}$ is not planar because it contains $K_{3,3}$ (if you didn't remember this, you might guess it is not planar, just because it has so many edges). You can't make 38 cents from 7 and 9. This one requires some experimentation, but if you do it systematically, it doesn't take long.

7) 201. We learned that $n = |V| = mi + 1 = 301$ and $l = n - i = 301 - 100 = 201$ (or, you could use the shortcut $l = (m - 1)i + 1$ if you memorized that).

This question was not really about memorizing obscure formulas. Even if you forgot them, you might re-discover the patterns by drawing a small tree with 2 or 3 internal vertices. Of course, you DO need to know the vocabulary ! And some formulas (see 5a) are worth memorizing.

8) $w\bar{x} + \bar{y}\bar{z} + \bar{w}xyz$. Many people omitted the second table (see page 839) but overall the results on this one were very good.

9) [partial answer] Suppose A has at least 6 friends. Apply Example 13 (done in class) to conclude these 6 contain 3 m.e's or 3 m.f's. Explain why we are done either way. I leave the other case to you, that A has at least 4 enemies. Explain why at least one case occurs. The results on this one were poor; I expected more people to remember this one. It was HW 6.2.27, and is similar to Ex 13, but with a bit more casework.

Bo) See the text.