Ch.7 HW

As of June 10, the only Ch.7 HW problems from our book are 1, 3, 4, 5, 6 (and 13 is below). You should also practice the main algorithms shown in class. Here are some additional homework problems related to matchings. I have listed the easiest ones first.

1) Draw a 3-regular graph G with a cut-vertex and a perfect matching.

2) Draw a non-Hamiltonian G with two disjoint perfect covers.

3) Prove: If G is connected and $p = 4$ and it is not $K_{1,3}$, then it has a perfect matching.

4) Use the K.E. Theorem ($\alpha = \beta_1$) to prove Hall's Theorem.

I am not yet sure how hard the next ones are, and would not recommend spending more than an hour per problem without asking for help.

5) The Petersen graph does not have two disjoint perfect covers.

6) If G is bipartite then $\beta_1(G) \geq |E|/\Delta(G)$.

Some hints or partial answers to the new problems (not in Gould's book):

1) If v is the cut one, there must be an edge $uv \in M$. Attach something like K_4 (minus an edge) to u (this will include two edges in M). Attach a similar subgraph to v.

2) Connect two C_4's by adding a edge.

3) G must contain a path of length 2. If it contains a path of length 3, we are done.

4) Assume 'neighborly' and that C is a cover. ETS that $|C| \geq |X|$ (with KE, this implies max $|M| \geq |X|$ and done). Split the cover into $C_X \cup C_Y$. Look for an inequality of the form $|C_Y| \geq$ something.

Some hints or partial answers to the ones in Gould's book:

1) Use induction. Roughly, $Q_{n+1} = Q_n \cup Q_n \cup$ some edges you don't need.

3) Assume the p_i are non-increasing. Two necessary conditions are that

a) $P = \sum p_i$ must even.

b) $2p_1 \leq P$.

Are these also sufficient ?

4) Hopefully easy.

5) At most one (start at a leaf; some edge in M must cover it, remove that edge, repeat). Probably you should phrase this as an induction proof. I think there are several careful solutions to this one online.

6) Should be easy. If not, practice on simpler examples, or see me.