Proof of Thm 1.5.1

I believe the proof in the text is wrong (though it also appears in other books) starting near the top of page 21. If you can find the error in the text proof without my help, I may give you some extra credit. Condition 1 about $V(G)$ is OK, but I will revise Condition 2. Here is my proof, starting on line 2 of page 21:

2. $I = |L \cap N(x_1)|$ is maximal, where $L = \{x_2 \dots x_{1+d_1}\}$.

Note that $|L| = |N(x_1)| = d_1$. If $I = d_1$ then $L = N(x_1)$ and the proof is easy because removing x_1 creates a graph that matches S_1. (see the last two sentences of the textbook proof; if this is not clear, you might want to review the first part of the proof, page 20, because we are just reversing that process). So, ETS that $I = d_1$ for the G defined by properties 1 and 2.

The main idea is that if $I < d_1$ then we can revise G (by *interchanging*) to increase I. But this would contradict property 2, that G maximizes I. The textbook description of *interchanging* on page 21 is correct. The next sentence, about 7 lines down, which begins 'However, in $H \dots$' should be rephrased to this:

However, in H, $N(x_1)$ now includes x_j (rather than x_i). Since $x_j \in L$ and $x_i \notin L$ (these remarks are implicit in the textbook proof, but they probably should have been stated more clearly) we have increased I by the interchange.

This completes the proof. Since I am asking you to learn a correct proof of this theorem, and my proof is the only completely correct one that I know of, you should learn this one. I am currently discussing all this with Prof Ramsamujh. If I am wrong about any of it, I will try to let you know well before the exam.

Question about DFSA

A student asked whether the DFSA always produces a tree of maximal height (eg it always locates the longest possible path away from x). This is an interesting conjecture, but it seems the answer is No. Here is a counterexample; basically a K_3 with one more vertex attached.

Let $V = \{x, a, b, c\}$ with 4 edges xa, ba, xb, bc. Please draw a picture for yourself. The DFSA might go like this: xb, bc (backup to to b) ba. This creates a tree with height 2. It does not necessarily notice the longer path xa, ab, bc, which would create a taller tree.