MAD 3305 - GRAPH THEORY

SUMMER A 2014

PREP FOR TEST #1

PROF HUDSON

KEY DEFINITIONS AND MAIN CONCEPTS

Digraph, in-degree & out-degree, Graph, multi-graph, pseudo-graph, degree of a vertex, degree sequence, graphical sequences, sub-graphs, induced sub-graphs, regular graphs, adjacency matrix; geometric, set-theoretic, and matrix representation; walk, trail, circuit, cycle, path, distances in weighted graphs; inaccessible vertices, eccentricity, diameter, center, & radius of a graph; connected graphs, the connected components, weakly & strongly connected digraphs, bridge, edge connectivity, vertex connectivity, cut-vertex, bridge, blocks, pendant vertices, trees, non-identical trees, leaves, minimum spanning trees, rooted trees, levels, height of a tree, children, parent.

MAIN ALGORITHMS

- 1. Graphical Sequence Algorithm & Graph Recovery Algorithm (Ch 1.5)
- 2. BFS Distance Algorithm (with proof)
- 3. Dijkstra's Distance Algorithm
- 4. DFS Connectivity Algorithm
- 5. Kruskal's and Prim's Algorithms (with proof for Prim).

MAIN THEOREMS (usually including a proof or explanation)

- 1. The decreasing sequence $\langle a, d_2, d_3, ..., d_p \rangle$ is graphical if and only if $\langle d_2-1, d_3-1, ..., d_{a+1}-1, d_{a+2}, ..., d_p \rangle$ is graphical (Graphical Sequence Theorem).
- 2. A connected graph with p vertices has at least p-1 edges.
- 3. If G is a disconnected graph, then G^c must be connected.
- 4. The number of walks of length n from v_i to $v_j = A^n[i,j]$ (no proof required).
- 5. G is a tree if and only if there is exactly one path between any two vertices.
- 6. A connected G is a tree iff |E| = |V|-1.
- 7. $k(G) \leq k_1(G) \leq \delta(G)$.

Notes: 1) You should also know the basic ideas of various lesser theorems covered in class, but not necessarily their proofs. 2) Thanks to Prof Ram for his template, which this page is based on.