MAD 3305 June 6, 2014
Exam II Prof. S. Hudson

NAME

Show all your work and reasoning for maximum credit. Do not use a calculator, book, or any personal paper. You may ask about any ambiguous questions or notation, or for extra paper. Hand in any extra paper you use along with your exam. The exam should take about one hour, to be followed by a lecture. Most problems are 10 points each, unless labeled.

1.a. How many non-identical spanning trees does the graph below have ?

1.b. Give an example (draw it) of a graph G with exactly $16 \cdot 5^3$ non-identical spanning trees.

2. (15pts total) 2.a. Prove that the n-cube Q_n is Hamiltonian, for $n \geq 2$.

2.b. Give an example of a $K_{n,m}$ with $n, m \geq 3$ that is neither Hamiltonian nor Eulerian, and justify your answer.

3. (20pts total) Answer each with True or False:

An arc e of a network is saturated if $f(e) = c(e)$.

If G has adjacency matrix A and degree matrix D then all the co-factors of $D - A$ are equal.

In every planar graph, $q \leq 3p - 6$.

A Huffman tree for a list of n frequencies has the minimum height among all full binary trees with n leaves.

If the FF Alg is applied to a network with 10 non-identical paths from s to t, then it will finish within 100 steps (augmentations).

4. Choose ONE proof (roughly the one given in class, or see me):

4a. State and prove the TFAE theorem about Euler circuits in G.

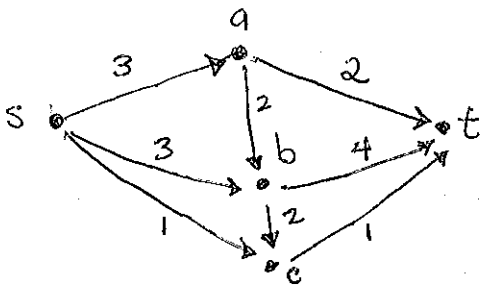
4b. State and prove Ore's theorem about Hamiltonian graphs.

5. Show that if G is a connected planar (p,q) graph with girth $g(G) = k \geq 3$ then $|E| \leq k(p-2)/(k-2)$. The girth is the shortest cycle length.

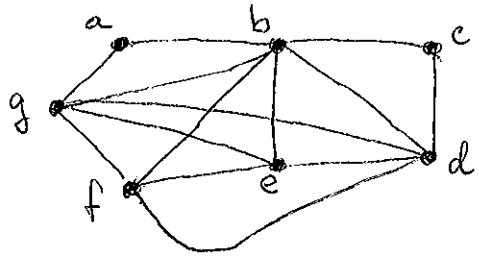
6. State the Chinese postman problem.

7. (15pts total) 7.a. Apply the FF algorithm to find the maximum flow F from s to t in this network. Start with a flow of 2 on the augmenting path s,a,b,t. Show each step clearly (in sentences, and/or by re-drawing the network). If you use extra paper, leave a note here.

7.b. Find a cut with capacity equal to F, the result of 7.a.



8. Apply the DMP algorithm to determine whether G below is planar. Draw each H_j. Explain your conclusion briefly.



Bonus (about 5 points): Draw the Icosian Game and solve it.