MAC 2311 | 11AM, Jan 30, 2009
Exam I | Prof. S. Hudson

1) [30 pt] Compute the limit. You may answer with '$+\infty$' or '$-\infty$' but not with 'd.n.e'. Show enough work or reasoning.

a) $\lim_{x\to-\infty} \frac{e^x+e^{-x}}{e^x-e^{-x}} =$

b) $\lim_{x\to+\infty} \frac{\sqrt{5x^2-2}}{x+3} =$

c) $\lim_{y\to\pi/2^-} \ln(\tan(y)) =$

2) [20pt] Short answer problems.

a) Solve for x, given that $\frac{1}{3-x} < 2$.

b) Find a formula for the area A of a circular sector in terms of its internal angle θ and the radius r (we used it in the $\sin(x)/x$ proof).

3) (20pts) Answer True or False. You do not have to explain.

a) $\cot(x)$ is continuous on $[-\pi/4, \pi/4]$.

b) $\frac{\cos(x)}{\ln(x)}$ is continuous on $[\pi/4, \pi]$.

c) $\lim_{x\to0} \frac{\sin(x^2)}{x^2} = 1$.

d) $\forall\epsilon > 2, \exists\delta > 3, \delta < \epsilon$.

e) $\forall\epsilon > 4, \exists\delta > 3, \delta < \epsilon$.

f) $\forall\epsilon > 0, \exists\delta > 0$, such that if $|x-1| < \delta$ then $|2x-2| < \epsilon$.

g) $\lim_{x\to a} f(x)$ exists if and only if $\lim_{x\to a^+} f(x)$ exists.

h) If $|x-2| < 1$ then $|2x-4| < 3$.

i) If $|x-23| < 2$ then $|x-22| < 3$.

j) The graph of $x = \cos^2(t)$, $y = \sin^2(t)$ is a straight line.

4) [15pt] Approximate the solution to $x^3+x^2-x+1=0$ within 0.1, with some explanation of your reasoning. You can use the data below instead of a calculator (a little arithmetic and organization is left for you).

x	x^3+x^2
1	2
-1.6	-1.536
-1	0
-2	-4
-1.8	-2.592
-1.4	-0.784

5) [15pts] Choose ONE of the problems below to do. Remember to use enough words and sentences - not just formulas.

a) Show that $\lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$. (you may use the other main trig limit).

b) Show that $\lim_{x\to 3} 4x-1 = 11$ using the definition of limit.

Bonus [5pts]: Use the definition of limit to prove that $\lim_{x\to 2} 1/x = 1/2$. This should be similar to the limit problem with x^2 we did in class (though the algebra will be a little different). If you answer on the back, leave me a note here.

Remarks and Answers: The average was approx 50/100, which is of course very low. The worst results were on the PreCalc question (#2) and the IVT problem (#4). The new [unofficial] scale is:

A's = 70 to 100
B's = 60 to 69
C's = 50 to 59
D's = 40 to 49
F's = 00 to 39

1a) -1. Each part of problem 1 was worth 10 points. For 1a), I gave 10 points for the correct answer with correct work:

$$\lim_{x\to-\infty} \frac{e^x+e^{-x}}{e^x-e^{-x}} \cdot \frac{e^x}{e^x} = \lim_{x\to-\infty} \frac{e^{2x}+1}{e^{2x}-1} = -1$$

I gave about 8 points for getting -1, using the method of 'dominant terms' (which usually works, but I advised against it, unless $f(x)$ is a rational function). If you ignored the minus sign in the $-\infty$, you probably got an answer of $+1$, and maybe 5 points partial credit, depending on your method.

1b) $\sqrt{5}$ with grading similar to 1a). For full credit, you need to multiply by $1/x$ on top and bottom (or some similar algebra).

1c) $+\infty$. This example is not hard to reason out, but it is harder to explain. I gave full credit for your reasoning if you included either

i) the graphs of both $\tan(x)$ and $\ln(x)$, or

ii) the formulas $\lim_{x\to\pi/2^-}\tan(x) = +\infty$ and $\lim_{x\to\infty}\ln(x) = +\infty$.

2a) $x < 5/2$ or $x > 3$ (of course, you must include both of these for full credit). The answer can be abbreviated $(-\infty, 5/2) \cup (3, \infty)$. There are several valid methods for getting this answer. My favorite is to locate the endpoints (such as $x = 5/2$) first. You can find them from

i) Set $|x - 3| = 1/2$ and get $x = 5/2$ or $x = 7/2$ (later, we see that this second value does not actually come into the answer, but it might do so in similar examples).

ii) Set $3 - x = 0$ to get $x = 3$.

Now, you can rely on a rough graph of $1/(3-x)$, or you can test each possible interval [such as $5/2 < x < 3$] by plugging in numbers [such as 2.6].

2b) $A = \theta r^2/2$ (explained in class).

3) FFTFT TFTTT I went over these after the exam. Most have simple explanations - see me if needed.

4) Several people didn't understand the question. The *solution* is the value of x that makes the equation true. You probably can't find it exactly, because this is a cubic polynomial, so you are asked to find a value *close* to the correct one. The method (based on the IVT - see Ch 2.5) is to plug in lots of x's until you can find two nearby ones that give different signs.

5) See the text or lectures notes. For a) use the conjugate.

Bonus: See me if you want help with this relatively hard problem. You can use $\delta = \text{Min}\,[1, \epsilon/1000]$ like the example I did in class, but the algebra is a little different. See the key to my PM exam for a similar proof.