MAC 2311 Sept 15, 2011
Exam I and Key Prof. S. Hudson

1) [15 pts] Short answer;

a) Find all values of θ such that $\cos(\theta) = -1/2$.

b) Find all values of x such that $|2x| < x + 1$.

c) Find a linear function f so that $f(1) = 2$ and $f(2) = 1$.

2) [35 pts] Compute each limit, showing enough work or reasoning. You may answer with a number, with $+\infty$ or $-\infty$. If none of these are correct, write 'd.n.e'.

a) $\lim_{x\to 2} \frac{x^2-4}{x-2} =$

b) $\lim_{x\to 0} \frac{\tan 4x}{\tan 5x} =$

c) $\lim_{x\to +\infty} \frac{x^2-4}{x-2x^2} =$

d) $\lim_{x\to 2^-} \frac{1}{x-2} =$

e) $\lim_{x\to 0} \frac{|x|}{x} =$

f) $\lim_{x\to -\infty} (1 - \frac{1}{x})^{-x} =$ Hint; substitute $t = -x$.

g) $\lim_{x\to 0+} \cot(x) =$

h)) $\lim_{x\to +\infty} \frac{5e^x}{e^x+25} =$

3) [20 pts] Answer True or False. You don't have to explain.

If $f(1) = 1$ and $f(2) = 2$ then there is some x so that $f(x) = 1.5$.

$f(x) = \sec x$ is continuous on $[-\pi/3, \pi/3]$.

If f is increasing ($f(x) = \tan^{-1}(x)$ for example), then always $m_{\text{sec}} \geq 0$.

The tangent line to a curve is a special type of secant line.

For every $\epsilon > 0$ there is a $\delta > 0$ so that if $|x| < \delta$ then $|2x - 1| < \epsilon$.

4) [10 pts] Find k so that this function is continuous;

$$f(x) = \begin{cases} kx^2, & \text{if } x \leq 2 \\ 2x + k, & \text{if } x > 2 \end{cases}$$

5) [10 pts] Note that $\lim_{x\to +\infty} \frac{1}{x^2} = 0$. Write out the definition of what this means using the usual letters, L and ϵ, etc. Find N (as small as you can) so that if $x > N$ then $|f(x) - L| < \epsilon$, with $\epsilon = 0.01$.

6) [10 pts] Choose ONE of the problems below to do. Include enough comments and/or justifications.

a) Prove that $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$, approximately as done in class.

b) Prove that $\lim_{x\to 3} x^2 = 9$ with the ϵ definition, approximately as done in class. [For partial credit you can change this problem to this easier proof; $\lim_{x\to 2} 4x + 1 = 9$.]

Remarks and Answers: Only about 15 out of 40 exams could be considered at all good. The average score of those was about 62 out of 100. Many people were weak on PreCalculus (Problem 1) and need to correct some deficiencies before moving on. Many people ignored my advice to study the Main Trig Limit proof (Problem 6). Several exam problems were straight from the HW, and should have been pretty easy. I don't see any evidence that any of the problems were super-hard; the results were a bit below 50% only on Problems 2fgh combined, and on 3 (TF). Here is an unofficial scale:

A's 75-100
B's 65-74
C's 55-64
D's 45-54

1a) $\theta = 2\pi/3 + 2n\pi$ or $\theta = 4\pi/3 + 2n\pi$, for some integer n. There are several ways to reason this out, from a picture or from identities, but first you need to have memorized that $\cos(\pi/3) = 1/2$.

1b) $x \in (-1/3, 1)$, or you can write this as $-1/3 < x < 1$. Again, there several good methods. I suggest solving these for x; set $2x = x + 1$, and then set $-2x = x + 1$, find the two endpoints. I'm leaving some of the thinking to you.

1c) $y = 3 - x$. Using a graph might be fastest. Otherwise, compute the slope $m = -1$ first, using rise over run (etc).

Note: If you scored below 8 out of 15 on Problem 1, then you have about a 1-in-10 chance of passing this course (take my pre-requisite quiz for more accuracy on this). Either drop, or plan to spend an extra 10 hours a week on PreCalc. You can see me, or one of our LAs, for help and advice.

2a) 4

2b) 4/5. But there are lots of bad ways to get this, which didn't get full credit. The good ways involve reducing to familiar limits, probably involving $sin(x)/x$ or something very close to that. For example, I'd accept

$$\lim \sin(4x)/x \cdot \lim x/\sin(5x) \cdot [\lim \cos(5x)\ /\lim \cos(4x)] = (4)(1/5)(1/1) = 4/5$$

2c) -1/2

2d) $-\infty$ I accepted almost any kind of reason, such as calculating $1/(1.99-2)$.

2e) D.N.E. Show the one-sided limits differ.

I wrote the sum of your 5 grades on page two at the bottom, just below 2e. You can check this, or just ignore it.

2f) e

2g) $+\infty$ because the denominator $\sin(x)$ approaches zero and the fraction is positive, since $x > 0$.

2h) 5

I realized too late that at 5 points each, problem 2 is actually worth 40 points, not 35. You can consider 2h to be a Bonus.

3) FTTFF See me for explanations.

4) 4/3. Set $\lim_{x\to 2^-} kx^2 = \lim_{x\to 2^+} 2x + k$ and get $4k = 4 + k$ etc. This is HW problem 1.5.29.b.

5) The def is $\forall\epsilon > 0, \exists N$ such that if $x > N$ then $|f(x) - L| < \epsilon$, but you can replace $\forall$ with words if you like, and replace $f(x)$ by $1/x^2$, L by 0. and can set $\epsilon = 0.01$.

Then an easy calculation gives $N = 10$.

This is essentially the assigned HW problem 1.4.41.

6a) This is proof in the book, and lectures, and in the announcement of Exam I.

6b) This is a pretty hard problem. Unless you followed the similar one I did in class, you shouldn't chose this one.

6b) (the easier alternate) I have up to 7 points for this, but there were no really good proofs. Some people followed the *discovery process* from the book [eg page 102] and calculated $\delta = \epsilon/4$, but then they didn't justify that. Some people used almost no words. I warned about this problem, and suggest using the style in my lectures instead.