MAC 2311 — Jan 31, 2014

Exam I and Key — Prof. S. Hudson

1) Solve for x, given $\log_{10}(\frac{x}{2} - 7) = 3$.

2) Which of these functions has a removable discontinuity ? Where is it ?

$$g(x) = \frac{|x-2|}{x-2}, \quad h(x) = \frac{x-4}{x-3}, \quad m(x) = \frac{x^2-2x}{x}$$

3) [15 pts] Compute $\lim_{x\to 0^+} \cot(x)$. Then, use the I.V.Thm to show that $\cot(x) - x = 0$ has a solution on $(0, \pi/2)$.

Compute each limit, showing enough work or reasoning. You may answer with a number, with $+\infty$ or $-\infty$. If none of these are correct, write 'd.n.e'.

4) $\lim_{x\to 2} \frac{x^2-4}{x^2+x-6} =$

5) $\lim_{x\to +\infty} \frac{x^4-4}{x-2x^3} =$

6) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} =$

7) [20 pts] Answer True or False. You don't have to explain.

$\lim_{x\to\infty} \sec^{-1}(x) = \lim_{x\to -\infty} \sec^{-1}(x)$

If $|x-1| < 3$ then $|x-2| < 2$.

If $|x-3| < 1$ then $|x-1| < 3$.

$f(x) = x^3 + x + 1$ has a root on $(0, 3)$.

For every $\epsilon > 0$ there is a $\delta > 0$ so that $\epsilon + \delta = 4$.

8) [15 pts] Choose ONE of the problems below to do. Include enough comments and/or justifications.

a) Prove that $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$, approximately as done in class.

b) Prove that $\lim_{x\to 3} x^2 = 9$ with the ϵ definition, approximately as done in class. [For partial credit, you can change this problem to $\lim_{x\to 3} 2x + 1 = 7$.]

Bonus) [5 pts, but this may be very hard]: What was wrong with the following 'proof' I gave in class that 1=0 ? Identify the first step that is wrong, and explain what's wrong with it.

$0 + 0 + \ldots = 0$ and $0 = 1 - 1$.

$1 - 1 + 1 - 1 + 1 - 1 + \ldots = 0$ and multiply by -1,

$-1 + 1 - 1 + 1 - 1 + 1 - \ldots = 0$ and substitute into the previous equation,

$1 + 0 = 0$.

Remarks and Answers: By popular vote, this was a 50 minute exam, and that seemed to be enough time for most people. However, we can always talk about this again, for the next exams.

The average was about 60 out of 100, based on the top 25 grades. The highest grades were 84 and 83. The worst results were on problem 3 (about 25% correct, including partial credit). The TF average was 60%, which is a bit low. The best results were on the straightforward calculation problems - 1, 4, 5 and 6 (about 75%). The unofficial scale is

A's 70 - 100
B's 60 - 69
C's 50 - 59
D's 40 - 49

To estimate how you are doing so far, you can average in your HW grades and your PreTest grade (with the same scale). But your Exam 1 grade will count much more at the end than these other factors. Answers:

1) 2014 (heh, heh!)

2) $m(x)$ does, at $x = 0$. Some people did a relevant calculation or two, but did not directly answer the question. I'd suggest checking all your answers when you are done with an exam, to make sure they are all there (circled if necessary) and in proper form.

3a) The limit is $+\infty$. If you have memorized the graph of $\cot(x)$ this is easy. If not, you can reason it out from $\frac{\cos(0.0001)}{\sin(0.0001)} \approx \frac{1}{\text{small}} \approx +\infty$. Several people got to the form $\frac{1}{0}$, which can usually be interpreted as $+\infty$ if there are no negative numbers involved. This is similar to the previous explanation.

3b) Part a) shows $f(x) = \cot(x) - x > 0$ for some x near 0. And $f(\pi/2) = 0 - \pi/2 < 0$. Since it is continuous on this interval, we can apply the IVT and deduce a zero. This problem is similar to the HW problem about polynomials of odd degree.

4) 4/5 by factoring.

5) Removing lower-order terms and canceling leads to $\lim -x/2 = -\infty$.

6) 1/6, by using the conjugate.

7) TFTFF

8) See the text or your lecture notes for model answers. I do not usually comment in detail about the grading of proofs, which is usually a bit unscientific, and may vary from time to time, but I will do so this here. There were a few basic categories of answers; about 1/3 of the answers were mostly bad. Perhaps these students were not prepared and drew a blank. Even so, I gave about 3 points partial credit for showing *something* relevant, such as the definition of limit for 8b, or the picture of the unit circle for 8a.

Among the better papers, about 40% chose 8a (the trig limit), with an average score of about 12 out of 15. I felt that most people memorized the proof, and understood perhaps 80% of it, which seems a fairly good statistic overall. A very common mistake (not very serious) was to say that the smallest area is $\sin(x)/2$; it should be $\sin(x)\cos(x)/2$.

Another 25% chose 8b (the harder nonlinear epsilon proof) with an average result of about 10 out 15. Most of the answers contained some good bits, such as $\delta = \epsilon/7$, but didn't quite hang together as complete organized proofs. These nonlinear examples are generally considered quite hard by most people at the MAC 2311 level.

Another 35% chose 8c (the easier linear limit) with an average result of about 8 out 10 (not 15). Since this problem was a bit harder than most of my Exam I proofs, I tried to give fairly generous partial credit, despite many typo's and gaps.

Overall, the scores on the proofs were similar to the scores on other problems. Among the prepared students, they were a bit higher. On later exams, they are usually a bit higher still, as people learn how to write them and how to study for them.

Bonus) The first mistake is the second line, $1 - 1 + 1 - 1 + 1 - 1 + \ldots = 0$, which is false. It would be OK with parentheses, like this $(1 - 1) + (1 - 1) + (1 - 1) + \ldots = 0$. With *finite* sums, the parentheses don't matter (by the associative property of addition). But with infinite sums, they are important - they cannot be omitted or moved around at will.

Valid methods for handling *infinite* sums like these are explained in MAC 2312. One of the key features is a limit. See me if you want to hear more ! It is not really very hard.