MAC 2311 AM, Feb 27, 2009
Exam II and Key Prof. S. Hudson.

1) (5pts each) Calculate the derivative of f (any valid method is OK. If you have simply memorized any of these, include a short note (I reserve the right to check you on this).

a) $f(x) = \sin(5x + 1)$

b) $f(x) = x \ln(x) - x$

c) $f(x) = \tan^{-1}(x) = \arctan(x)$

d) $f(x) = |3x|$

e) $f(x) = 2^x$

f) $f(x) = \cos^3(\sin(2x))$

2) (10pts) Compute the derivative using the definition (compute a limit): $y = 1/x$.

3) (10pts) Find the second derivative, $y''(\pi)$, where $y = \sec(2x)$.

4) (10pts) State the definition of *continuous at x=a*. Use this definition to prove (check) that $f(x) = x^3$ is continuous at $x = 2$. This should be quick - you can use any known facts about polynomials and limits (no epsilon stuff required).

5) (10pts) Use a local linear approximation to approximate $\sqrt{24}$.

6) (10pts) A 13 foot ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 4 ft/sec, how fast will the top of the ladder be moving down the wall when it is 12 ft above the ground? [note: $12^2 + 5^2 = 13^2$]

7) (10pts) Compute the limit using LHR

$$\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{e^x - 1} =$$

8) (10pts) CHOOSE ONE. Remember to explain everything;

A) State and prove the usual formula for $(f^{-1})'(x)$.

B) State and prove the Power Rule (but only the first version, for $n = 1, 2, 3 \ldots$).

Bonus (5pts) - Find a solution to the DE: $y' = 5y$ (this may have to be guesswork, unless you remember a certain example from class).

Remarks: The average was about 68, which is OK, fairly normal. You can use the original scale, on my syllabus, to get your letter grade for this exam [so A's start around 81, etc]. The average grades on most problems were OK, except for problem 4, the definition of *continuous*. I did not explicitly tell you to memorize this. But in general you should learn the main definitions, and the statements of the theorems in a math class. This is one of the best ways to learn math (in addition to HW, of course) and it becomes even more important as you get into more advanced courses.

1a) $5\cos(5x+1)$

1b) $\ln(x)$

1c) $\frac{1}{1+x^2}$

1d) 3 if $x > 0$, and -3 if $x < 0$ (dne at 0).

1e) $\ln(2) \cdot 2^x$

1f) $-6\cos^2(\sin(2x))\sin(\sin(2x))\cos(2x))$

2) $\lim_{h\to 0}(\frac{1}{x+h} - \frac{1}{x})/h = \ldots = -1/x^2$

3) 4. Use the Chain Rule twice, the Product Rule, $\tan(2\pi) = 0$ and $\sec(2\pi) = 1$.

4) $\lim_{x\to a} f(x) = f(a)$ (even better, include comments as in the text). For this example, we check that $\lim_{x\to 2} x^3 = 2^3$. This follows from theorems on limits in Ch 2.

5) $5 + \frac{1}{2\sqrt{25}}(-1) = 4.9$

6) Draw, label variables, get $xx' + yy' = 0$, etc and $y' = -20/12$. So, it's moving downwards at 5/3 ft/sec.

7)

$$\lim \frac{\frac{1}{2\sqrt{x+4}}}{e^x} = 1/4$$

8) See the text. Of course, part A is easier, but it wasn't advertised.

Bonus: I hoped you'd remember the example $y = e^{5x}$. I didn't notice until after the exam that $y = 0$ is a much simpler answer (but nobody else did either).