MAC 2311 PM Feb 27, 2009
Exam II and Key Prof. S. Hudson

1) [30 pts] Compute the derivative, $f'(x)$ (shortcuts such as the Product Rule are OK for these, but show all work);

a) $f(x) = x\sin(x^2)$

b) $f(x) = \tan^{-1}(2x)$

c) $f(x) = \frac{\sqrt{x+1}}{x}$

d) $f(x) = \ln(\ln(x))$

e) $e^{\sec(x)}$

f) Compute the second derivative, $f''(x)$, given that $f(x) = \sin(3x+1)$.

2) [10 pts] Find the derivative using logarithmic differentiation: $y = x^x$.

3) [10 pts] A 6-foot man is walking towards a 15-foot lamppost at 3 feet per second. How fast is the length of his shadow decreasing ?

4) [10 pts] Find the slope of the tangent line to $y = x^2$ at $x = 4$, using the definition of m_{tan}. So, you will need to compute a limit. Do not use any methods beyond Ch 3.1, such as derivative shortcuts.

5) [15 pts] Answer True or False. You do not have to explain.

For all x, $\ln(e^{5x}) = 5x$.

If f is differentiable, then f' is continuous.

[using notation from the LLA section] $\triangle y = \frac{dy}{dx}dx$.

If f is continuous (on its domain), then f^{-1} is continuous (on its domain).

Every continuous function is differentiable.

6) [10pts] Compute these limits from Ch 2.6. Do NOT use LHopital's rule:

a) $\lim_{x \to +\infty} \cos(1/x)$

b) $\lim_{x \to 0} \frac{e^{3x}-1}{x}$ Hint: You can use the fact that $\lim_{x \to 0} \frac{e^{x}-1}{x} = 1$. Now, find a substitution.

7) [15 pts] Choose ONE of the problems below to do. Include enough comments and/or justifications.

a) State and prove the Power Rule (when $n > 0$ is an integer).

b) Show that a polynomial of odd degree must have a root.

Bonus: [5 pts] Prove the Quotient Rule as done in class.

Remarks and Answers: The average was about 58 / 100. The worst results were on the related rates problem (from the HW!) and on the proof. The rest were generally OK. The highest grade was approx 80, as on E1.

The (unofficial) scale is:
A's = 74-100, B's = 63-73, C's = 52-62, D's = 41-51.

1a) $\sin(x^2) + 2x^2 \cos(x^2)$ [Product Rule]

1b) $\frac{2}{1+(2x)^2}$ [Chain Rule]

1c) $(\frac{x}{2\sqrt{x+1}} - \sqrt{x+1})/x^2$ [Quotient Rule]

1d) $\frac{1}{\ln(x)\ x}$ [Chain Rule]

1e) $e^{\sec(x)} \sec(x) \tan(x)$ [Chain Rule]

1f) $-9\sin(3x+1)$ A few people wrote this incorrectly as $9 - \sin(3x+1)$.

2) $y' = (\ln(x)+1)x^x$

3) 2 ft/sec. The results were awful on this one, even though it was a HW

problem. I drew the picture on the board, upon request, but most people didn't get the next step, of labeling the variables. For example, the length of the shadow could be labeled s, so the question is to find ds/dt, etc.

4) $\lim_{h\to 0} \frac{(4+h)^2-4^2}{h} = \ldots = 8$

5)TFFTF

6a) 1

6b) 3 (replace $3x$ by t, for example)

7) See text.

Bonus: $(f/g)' = (f \cdot 1/g)' = f' \cdot 1/g + f \cdot (-g'/g^2) = \frac{f'g-fg'}{g^2}$