MAC 2311 — Oct 13, 2011

Exam II Key — Prof. S. Hudson

1) [30 pts] Compute the derivative, $f'(x)$ (shortcuts such as the Product Rule are OK for these, but show all work);

a) $f(x) = x^3 \cos(4x)$

b) $f(x) = 3^{3x}$

c) $f(x) = \frac{2x+1}{x}$

d) $f(x) = \ln(e^{\sin(x)})$

e) $\sin^{-1}(3x)$

f) Compute the second derivative, $f''(x)$, given that $f(x) = \sin(3x + 1)$.

2) [10 pts] Find the derivative dy/dx using implicit differentiation: $x^2y + 3xy^3 - x = 3$.

3) [10 pts] Find the equation of the tangent line to $f(x) = \sqrt{x}$ at $x_0 = 25$. Use this LLA to estimate $\sqrt{26}$.

4) [10 pts] Find the slope of the tangent line to $y = 1/x$ at $x = 4$, using the definition of m_{tan}. So, you will need to compute a limit. Do not use any methods beyond Ch 2.1, such as derivative formulas.

5) [15 pts] Answer True or False. You do not have to explain.

For all x, $\sin(\sin^{-1}(x)) = x$.

If f is differentiable, then f is continuous.

The function $|x|$ is defined implicitly by $(x - y)(x + y) = 0$.

If f is continuous and has an inverse, then f^{-1} is also continuous.

If f is continuous, there is at least one point x in its domain where f is differentiable.

6) [15 pts] Compute these limits from Ch 1.6. Do NOT use LHopital's rule:

a) $\lim_{t\to 0} \frac{t^2}{1-\cos(t)}$

b) $\lim_{x\to 0} e^{\cos(x)}$

7) [10 pts] Choose ONE of the problems below to do. Include enough comments and/or justifications.

a) State and prove the Power Rule (when $n > 0$ is an integer).

b) State and prove the Product Rule.

c) Finish this formula in the usual way and prove it; $\frac{d}{dx} \tan^{-1}(x) =$.

Bonus: [5 pts] Choose ONE; you may continue on the back, but leave me a note.

a) Prove the Reciprocal Rule as done in class (without using the Quotient Rule).

b) In problem 3), decide if your approximation is too high (more than $\sqrt{26}$) or too low, without actually calculating numbers. Explain your decision in terms of graphs and/or properties of $f(x) = \sqrt{x}$, ideally including f''.

Remarks and Answers: The average was about 70 / 100, which is pretty normal. The grades were generally good, except for problems 5 and 6 (TF and limits), for which they were approx 50%. The unofficial scale is

A's 80 - 100
B's 70 - 79
C's 60 - 69
D's 50 - 59

1a) $y' = 3x^2 \cos(4x) - 4x^3 \sin(4x)$ from the Prod.Rule.

1b) $y = 9^x$, so $y' = \ln 9 \cdot 9^x$ based on a memorized formula. Or, you could use Log-Diff.

1c) $y = 2 + 1/x$, so $y' = -1/x^2$. The Quotient Rule is also OK, but longer, therefore riskier.

1d) $y = \sin x$, so $y' = \cos x$. The Chain Rule is also OK, but longer, therefore riskier.

Note that I added your grades on the problems above and wrote it on page 1. I added the next two parts separately, with that sum written on page 2.

1e) $3/\sqrt{1 - 9x^2}$

1f) $-9 \sin(3x + 1)$

2) $y' = (1 - 2xy - 3y^3)/(x^2 + 9xy^2)$ (see 3.1.5)

3) The eqn of the TL is $y = 5 + (x - 25)/10$; other forms OK. If you wrote down any other equations, this one should be circled or clearly labeled. I did not give full credit for the general formula $f(x_0) + f'(x_0)(x - x_0)$, nor for $5 + (1/10)(26 - 25)$, which is not the equation of a line. But it does answer part b), $\sqrt{26} \approx 5.1$.

You were not required to use the limit definition in this problem, but you were in the next one.

4) In brief,

$$\lim_{h\to 0} \frac{1/(4+h) - 1/4}{h} = \lim_{h\to 0} \frac{-h}{4(4+h)h} = -1/16$$

No major shortcuts allowed, but you can check the answer from $-1/x^2|_{x=4} = -1/16$.

Several people got $-1/16$ without computing the limit correctly. Though it is hard to say exactly how they did this, it certainly appears dishonest, and should be avoided. This kind of answer got about 5 out of 10 points, whereas minor calculation errors leading to $+1/16$, for example, got more.

5) FTTTF. As always, see me for explanations. Part 5 would be hard, except that we discussed it in class. Part 3 is from the exercises (see 3.1.23); you may need to re-read the Definition (page 186).

6a) [8pts] 2; multiply on the top and bottom by $1+\cos(t)$ and simplify to $\lim 1+\cos(t) = 2$.

6b) [7pts] $e^1 = e$, by continuity.

7) See the text or lecture notes. After grading 7a, it seemed many people ignored the instructions. A few remarks on that:

"State it" means you should write $\frac{d}{dx}\, x^n = nx^{n-1}$, including BOTH sides, at the start of your answer.

"Include enough comments" implies you should include (at the very least) "the definition of derivative" and "by the binomial theorem" in your proof.

Similar remarks apply to 7b and 7c, though 7c probably requires less explanation.

Bonus a) Use the Chain Rule; see notes. b) It is too high. A graph shows the tangent line lies *above* the curve in this example. That's because the graph curves downwards, which is because $f'' < 0$ (we'll come back to this, in more detail, in the next chapter).