MAC 2311 Feb 27, 2014
Exam II and Key Prof. S. Hudson

1) [20 pts (corrected from 30 pts)] Compute the derivative, $f'(x)$ and show all work;

a) $f(x) = \sin(x^3)$

b) $f(x) = (\ln x)^{\tan x}$ [suggestion: log-diff'n]

2) [10 pts] Find the second derivative of $f(x) = \sec(x)$.

3) [10 pts] Find $\frac{dy}{dx}$ given that $\sin(xy^2) = x$.

4) [15 pts] Let $f(x) = \tan(x)$ on $(-\pi/2, \pi/2)$. Explain why f must have an inverse function. Then compute and simplify $\frac{d}{dx}\tan^{-1}(x)$ using the $(f^{-1})'$ theorem as done in class.

5) [10pts] An circular oil spill is spreading at a rate of 500π square feet per hour when the radius is 100 feet. How fast is the radius increasing then?

6) [10pts] Compute $\lim_{x\to+\infty} \frac{x^2+2x}{x+1} - x$. Use L'Hopital's Rule if / when possible.

7) [15 pts] Answer True or False. You do not have to explain.

The domain of f' is the same as the domain of f.

$\frac{d}{dx}\tan^{-1}(x) + \frac{d}{dx}\cot^{-1}(x) = 0$.

For all x, $e^{\ln(5x)} = 5x$.

Every continuous function is differentiable.

If f is continuous and has an inverse, then f^{-1} is continuous (on its domain).

8) [10 pts] Choose ONE of the problems below to do. Use the definition of derivative. Include enough comments and/or justifications.

a) State and prove the Power Rule (when $n > 0$ is an integer).

b) Show that $\frac{d}{dx}(\cos(x)) = -\sin(x)$.

Bonus: Give examples of two functions f and g which are not continuous, but $f \circ g$ is continuous.

Remarks: The average of the top 20 scores was about 64, with high scores of 97 and 87. The highest average scores were on problems 2 and 8 (about 80% correct) and the lowest scores were on 4 and 6 (about 50% correct). I did not compute 1a and 1b separately, the scores on 1a seemed very high and the ones on 1b very low. The rough scale for Exam 2 is

A's 74 - 100
B's 64 - 73
C's 54 - 63
D's 44 - 53
F's 0 - 43

I also computed your current semester average, including the two exams at 20 points each and the PreTest at 5 points. I will include your HW later, of course (also any grading adjustments, projects, make-ups, etc). See the upper right corner of your exam, page 1. The average for the same 20 students on this stat was about 60, with highs of 92 and 78. The rough scale for these numbers is 4 points (from 64-60=4) lower than the one above:

A's 70 - 100
B's 60 - 69
C's 50 - 59
D's 40 - 49
F's 0 - 39

The drop date is (I think) March 17, and I leave this to you. But, if your semester average is below about 43, you don't have much chance to pass with a C. If you are in the 44-52 range, you probably will not get a C by coasting, but you do have a realistic chance to improve your work habits and your grade in time to pass. Note that $\pm$'s will be used, so the lowest C is currently about 53, not 50.

Remarks on the PreTest + PreCalc HW: Practice with Precalculus seems to have helped some people significantly. On the whole, the semester grades now seem about 7 points lower than the scores on the PreTest. I have records of 7 people who scored low on the PreTest and did the Precalculus HW. These people raised their grades about 14 points on average since then (though, to be honest, most of these people are not passing yet).

Answers:

1a) $f'(x) = 3x^2 \cos(x^3)$, Chain Rule.

1b) $f'(x) = (\ln x)^{\tan x}[\sec^2 x \ln x + \frac{\tan x}{x \ln x}]$. Review Log-Diff if needed; take ln of both sides and then a derivative of both sides.

2) $f'(x) = \sec x \tan x$ so $f''(x) = \sec^2 x \tan x \sec^3 x$, Product Rule.

3) $\frac{dy}{dx} = \frac{1-y^2\cos(xy^2)}{2xy\cos(xy^2)}$, Implicit Diff. A few people started by trying to solve for y (using $\sin^{-1}$, etc), which is not totally crazy, but it did not turn out very well.

4) The function is increasing because $f' = \sec^2(x) > 0$. So, it is one-to-one, and must

have an inverse. For any credit on the explanation, you must at least mention one-to-one or the horizontal line test. No credit for remarks about continuous or differentiable. The derivative is

$$\frac{1}{f'(f^{-1})} = \frac{1}{\sec^2(\tan^{-1}(x))} = \frac{1}{1+x^2}$$

5) $A = \pi r^2$, $A' = 2\pi r r'$, $500\pi = 2\pi 100 r'$, $r' = 5/2$ ft/hour.

6) 1. Begin by getting a common denominator and changing the problem to $\lim \frac{x}{x+1} = 1$ (by LHR). A fairly common error was lazily omitting the "lim" after the 1st or 2nd step, and then stopping at $\frac{x}{x+1}$ without ever taking a limit. Try to build better habits than this while doing the HW!

7) FTFFT, feel free to come by to discuss these.

8) See the text or lecture notes. Most people chose (a). The most common problem was a lack of explanation. There is some room for differences in style, but a good proof should include phrases like these:

$\frac{d}{dx}x^n = \cdots$ (at the start of the calculation)

by the definition of derivative

by the Binomial Theorem

a brief comment about the middle terms (ones with h^k as a factor, $k \geq 2$).

And, why these disappear later.

Bonus: Choose g to be any discontinuous function with range in $[-1, 1]$ (such as $g = x/|x|$). Choose f to be constant on $[-1, 1]$ but discontinuous somewhere else (such as $f = (x-3)/|x-3|$). Then $f \circ g$ is constant, so it is continuous.