MAC 2311 April 5, 2007
Exam III and Key Prof. S. Hudson.

1) (10pts) A spherical balloon is inflated so that its volume ($V = 4\pi r^3/3$) is increasing at a rate of 3 ft^3/min. How fast is the diameter increasing when the radius is 1 ft.?

2) (10pts) Compute each limit (and show work);

a) $\lim_{x\to+\infty}(1-2/x)^x$

b) $\lim_{x\to 0} \frac{\tan(x)}{\tan^{-1}(x)}$

3) (25pts) Compute:

a) Find dy/dx: $y = e^{1/x}$

b) Find dy/dx: $y = \tan^{-1}(x^3)$

c) Find dy/dx using logarithmic differentiation: $y = x^x$

d) Find dy/dx using implicit differentiation: $x^2 + y^2 = 100$

e) Find d^2y/dx^2 using implicit differentiation: $2x^2 - 3y^2 = 4$

4) (15 pts) Give an example of a function $f(x)$ for each part, or explain why none exists. [Give three separate answers].

a) It has a stationary point at $x = 0$, but that point is not a relative extrema.

b) It has a relative extrema at $x = 0$, but that point is not a stationary point.

c) $f''(0) = 0$ but $x = 0$ is not an inflection point.

5) (10pts) Prove that $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$ using a theorem about inverse functions (or the chain rule).

6) (10pts) Answer True or False:

$\tan^{-1}(x)$ has a maximum value on $(-\infty, +\infty)$.

Every quadratic polynomial has exactly one critical point.

If $f'(x) < 0$ on (a,b), then f is decreasing on (a,b).

If f has only one relative extrema on I, and it is a relative minima, then it is also an absolute minima.

If f is continuous on (a,b) and $\lim f = -\infty$ as $x \to a^+$ and as $x \to b^-$ then f has an absolute max but no absolute min on (a,b).

7) (10pts) Sketch a graph of $f(x) = 1/(x^2 - 1)$ paying special attention to any asymptotes and critical points. Label it carefully. Hint: $f\ '(x) = \frac{-2x}{(x^2-1)^2}$. Draw a number line for f' (but you can omit f").

8) (10pts) A rectangular area of 3200 ft^2 is to be fenced off. Two opposite sides will use fencing costing \$ 1 per foot and the remaining sides will cost \$ 2 per foot. Find the dimensions of the rectangle with least cost.

Bonus (5pts): Describe carefully what the three cases are, in the proof of Rolle's Theorem. You do not have to prove them, but explain why they include all possible "nice" functions on (a,b).

Remarks and Answers: The average was about 58, so the unofficial scale is adjusted about 10 points: A's = 70 to 100, B's = 60 to 70, C's = 50 to 60, D's = 40 to 50, F's = 00 to 40. I also computed the sum of your 3 exam scores and gave you an estimate of your current semester grade (based only on that data, so far).

1) $dD/dt = \frac{3}{2\pi}$. Start with $3 = dV/dt = 4\pi r^2 r'$ and get $r' = \frac{3}{4\pi}$. Then use $D = 2r$, etc. As usual, most people who took a derivative with respect to t did OK. And that should be obvious in a related rates problem, but many people didn't do it.

2a) e^{-2}, 2b) 1

3a) $dy/dx = -e^{1/x}/x^2$
3b) $dy/dx = 3x^2/(1 + x^6)$
3c) $dy/dx = (\ln(x) + 1)x^x$
3d) $dy/dx = -x/y$
3e) $d^2y/dx^2 = (18y^2 - 12x^2)/(27y^3)$

4) It can be difficult to think of examples, but these are fairly simple ones we covered in class; $f(x) = x^3$, $f(x) = |x|$ and $f(x) = x^4$ (of course, other answers are possible, but these were the most common correct answers).

5) See the text or lecture notes. You should simplify $\cos(\sin^{-1}(x))$ using a triangle.

6) FTTTT

7) The graph looks a lot like Ch 5.3, Example 1. But the VA's are at $x = \pm 1$ and the HA is at $y = 0$. The stat point is at (0,-1).

8) The expensive sides should be 40 ft and the others 80 ft. Apparently several people got this by guessing, but I didn't give full credit unless you showed valid work (such as $3200 = A = xy$ and Cost $= 2x + 4y$, and a derivative later).

Bonus: Most people offered the entire proof of Rolle's theorem (not asked for), but didn't answer the actual question very carefully. Please see me if interested in this one.