MAC 2311 AM April 3, 2009
Exam III and Key Prof. S. Hudson

1) [30 pts] Compute the derivative, y'

a) $y = \sin^{-1}(x^2)$

b) $y = (\sin(x))^x$

c) $\sin(xy) = y$

2) [10 pts] A farmer has 200 yd of fence with which to construct 3 sides of a rectangular pen. An existing long, straight wall will be the fourth side (you can ask about a picture). What dimensions will maximize the area of the pen?

3) [10pts] Graph $f(x) = xe^x$ including any stationary points, inflection points or asymptotes. You can use this: $\lim_{x\to-\infty} xe^x = 0$. You can use $e \approx 3$ and $e^2 \approx 7$ to plot points (for example instead of $(1, e)$, you can plot $(1, 3)$).

4) [10 pts] Find the maximum and minimum values of $f(x) = x^3 - 3x^2 - 9x + 5$ on $[-2, 4]$.

5) [10 pts] Compute the limit $\lim_{x\to 0} \frac{\tan^{-1}(2x)}{3x}$

6) [20 pts] Answer True or False. You do not have to explain.

An absolute maximum must be a relative maximum.

If $f'(x) > g'(x)$ on $(1, 4)$, then $f(3) - f(2) > g(3) - g(2)$.

The graph of $y = x^{2/3}$ has a vertical tangent line at $x = 0$.

If f is differentiable on the interval (a, b) then f has a maximum value on (a, b).

If f is increasing on (a, b) then $f'(x) > 0$ there.

7) [10 pts] Choose ONE of the problems below to do. Include enough comments and/or justifications.

a) State and prove Rolle's Thm

b) State and prove the Mean Value Thm

Remarks and Answers: The average was about 60 out of 100. The worst results were on the TF and the proof [possibly, some people ran out of time]. The graphing problem scores averaged about 50%. The other problems were OK, and for the word problem the average was 85%. The rough scale for this exam is

A's = 75-100; B's = 65-74; C's = 55-64; D's = 45-54;

1a) $\frac{2x}{\sqrt{1-x^4}}$

1b) $[\ln(\sin(x)) + x\cot(x)](\sin(x))^x$

1c) $\frac{y\cos(xy)}{1-x\cos(xy)}$

2) 50 x 100, at the stat pt. Many people didn't bother to check the end-points, which is a bad habit, but it doesn't affect the answer here, and I didn't take off points this time. I did take off points for a similar omission on problem 4.

3) Stat pt at $(-1, -1/e)$. Inf pt at $(-2, -2/e^2)$. HA at $y = 0$ (LHS only). See me for the graph.

4) Stat pts at -1 (the max) and 3 (the min). Plug in these two, *and the endpoints*, to see which is the max and which is the min. There is no need to draw the number lines or apply a 1st/2nd deriv test. The info you get from those is used mainly for graphing (or when there is only ONE stat pt in some max/min problems).

Since the question asks about *values*, you should really answer in terms of y; The minimum value is $y = -22$ and the max is $y = 10$.

5) 2/3

6) FTTFF On the last one, it is not clear whether f is differentiable. Also, $y = x^3$ is increasing, but $f'(0) = 0$.

7) See the text or lecture notes.