1) [30 pts] Compute the derivative, $y'$

a) $y = \sin^{-1}(x^2)$

b) $y = (\sin(x))^x$

c) $\sin(xy) = y$

2) [10 pts] A farmer has 200 yd of fence with which to construct 3 sides of a rectangular pen. An existing long, straight wall will be the fourth side (you can ask about a picture). What dimensions will maximize the area of the pen?

3) [10 pts] Graph $f(x) = xe^x$ including any stationary points, inflection points or asymptotes. You can use this: $\lim_{x \to -\infty} xe^x = 0$. You can use $e \approx 3$ and $e^2 \approx 7$ to plot points (for example instead of $(1, e)$, you can plot $(1, 3)$).

4) [10 pts] Find the maximum and minimum values of $f(x) = x^3 - 3x^2 - 9x + 5$ on $[-2, 4]$.

5) [10 pts] Compute the limit $\lim_{x \to 0} \tan^{-1}(2x)/3x$.

6) [20 pts] Answer True or False. You do not have to explain.

   An absolute maximum must be a relative maximum.

   If $f'(x) > g'(x)$ on $(1, 4)$, then $f(3) - f(2) > g(3) - g(2)$.

   The graph of $y = x^{2/3}$ has a vertical tangent line at $x = 0$.

   If $f$ is differentiable on the interval $(a, b)$ then $f$ has a maximum value on $(a, b)$.

   If $f$ is increasing on $(a, b)$ then $f'(x) > 0$ there.
7) [10 pts] Choose ONE of the problems below to do. Include enough comments and/or justifications.

a) State and prove Rolle’s Thm

b) State and prove the Mean Value Thm

Remarks and Answers: The average was about 60 out of 100. The worst results were on the TF and the proof [possibly, some people ran out of time]. The graphing problem scores averaged about 50%. The other problems were OK, and for the word problem the average was 85%. The rough scale for this exam is

A’s = 75-100; B’s = 65-74; C’s = 55-64; D’s = 45-54;

1a) \( \frac{2x}{\sqrt{1-x^4}} \)

1b) \( [\ln(\sin(x)) + x \cot(x)](\sin(x))^x \)

1c) \( \frac{y \cos(xy)}{1-x \cos(xy)} \)

2) 50 \times 100, at the stat pt. Many people didn’t bother to check the endpoints, which is a bad habit, but it doesn’t affect the answer here, and I didn’t take off points this time. I did take off points for a similar omission on problem 4.

3) Stat pt at \((-1, -1/e)\). Inf pt at \((-2, -2/e^2)\). HA at \(y = 0\) (LHS only). See me for the graph.

4) Stat pts at -1 (the max) and 3 (the min). Plug in these two, and the endpoints, to see which is the max and which is the min. There is no need to draw the number lines or apply a 1st/2nd deriv test. The info you get from those is used mainly for graphing (or when there is only ONE stat pt in some max/min problems).

Since the question asks about values, you should really answer in terms of \(y\); The minimum value is \(y = -22\) and the max is \(y = 10\).

5) 2/3

6) FTTFF On the last one, it is not clear whether \(f\) is differentiable. Also, \(y = x^3\) is increasing, but \(f'(0) = 0\).

7) See the text or lecture notes.