MAC 2311 PM April 3, 2009
Exam III and Key Prof. S. Hudson

1) [20 pts] Ch 4.1-3, Compute the derivative, y'

$y = \log_2(3x^4)$

Use the f^{-1} theorem to compute y' when $y = \sin -1(x)$, and simplify.

2) [10 pts] Find the dimensions of the cylinder with maximal volume that can be inscribed in a cone with radius 6 and height 10. You may need to use similar triangles at some point (or maybe the $y = mx + b$ formula instead).

3) [10pts] Find the relative extrema of $f(x) = x^3(x + 1)^2$ and classify (as max/min).

4) [10 pts] Give an example of each. For maximal credit, give formulas (or, pictures are OK for partial credit).

a) A function with an absolute max which is not a relative max.

b) A function with a critical point which is not a stationary point.

5) [20 pts] Compute the limits using L'Hopital's Rule.

a) $\lim_{x\to\pi} \frac{\sin(x)}{x-\pi} =$

b) $\lim_{x\to\infty}(1 + 3/x)^x =$

6) [20 pts] Answer True or False. You do not have to explain. Do not make unstated assumptions (that f is positive, continuous, etc).

If f is continuous on $[2, 3]$ then it must have an absolute minimum on $[2, 3]$.

Since $\ln(x)$ and $\ln(\pi x)$ have the same derivative, they differ by a constant.

If $f(0) = f(10)$ then f must have a stationary point.

If f is differentiable on the interval (a, b) then f has a maximum value on (a, b).

If f is increasing on (a, b) then $f'(x) > 0$ there.

7) [10 pts] Choose ONE of the problems below to do. Include enough comments and/or justifications.

a) Prove that if $f'(x) < 0$ on (a, b) then f is decreasing there. Use the MVT.

b) State and prove the Mean Value Theorem.

Remarks and Answers: The average was about 45 out of 100. I am still deciding how to scale this, or whether to allow a second chance on this material.

1a) Simplify to $y = 4\ln(x)/\ln(2) + C$ and then get $y' = 4/(x\ln(2))$

1b) $y' = (1 - x^2)^{-1/2}$ (see text or lecture for the method).

2) Most people drew a decent picture, but many did not bother to label any variables (then the rest was usually confused). Label the cylinder with r and h and use $V = \pi r^2 h$. Get $h = 10 - 5r/3$ (from similar triangles or from $y = mx + b$). The rest is fairly routine; get $r = 4$ and $h = 10/3$.

3) It is probably easier to use the Product Rule than to multiply out, though either method is OK: $f'(x) = 3x^2(x+1)^2 + 2x^3(x+1) = x^2(x+1)(5x+3)$ which gives 0, -1 and -3/5. A number line (1st DT) shows -1 is a rel max, -3/5 is a rel min, and 0 is neither.

4a) $y = x$ on [0,1]. Giving examples can be hard, but we went over these two days before the exam.

4b) $y = |x|$

5a) -1

5b) e^3

6) FTTFF

7) See the text or lectures.