MAC 2311 Nov 10, 2011
Exam III and Key Prof. S. Hudson

1) [25 pts] Compute these limits using any valid methods, showing all work;

a) $\lim_{x\to\pi} \frac{\sin(2x)}{(x-\pi)\cos(3x)}$

b) $\lim_{x\to 0} \frac{\sin^{-1}(x)}{3\sin(x}$

c) $\lim_{x\to 0^+} \sqrt{x}\ln(1/x)$

d) $\lim_{x\to 0^+} x^{1/\ln(x)}$

e) $\lim_{x\to 0^+} x^{1/x}$

2) [10 pts] A farmer has 140 yd of fence with which to construct 3 sides of a rectangular pen. An existing long, straight wall will be the fourth side (you can ask about a picture). What dimensions will maximize the area of the pen?

3) [15pts] Sketch a graph of $f(x) = 2x^3 - 3x^2 - 36x + 5$ including any stationary points, inflection points or asymptotes. List the intervals of increase/decrease, and of concave up/down, as usual.

4) [10 pts] Find the maximum and minimum values of $f(x) = x - 2\sin(x)$ on $[-\pi/4, \pi/2]$.

5) [10 pts] A 7 ft. tall Texan is walking towards a 21 ft tall street lamp at a rate of 4 ft per sec. At what rate is his shadow length changing ? (you may ask for a picture)

6) [20 pts] Answer True or False. You do not have to explain.

If $f''(a) = 0$ then a is an inflection point of f.

An absolute maximum of f must be a relative maximum or an endpoint.

If f is a polynomial on the interval $[a, b]$, then f has a maximum value on $[a, b]$.

If $f'(x) < 0$ on (a, b) then f is decreasing there.

If $\lim_{x\to\infty} f(x) = 5$ then the graph of f cannot cross the line $y = 5$.

7) [10 pts] Let $f(x) = (x-3)^{1/5}$. Find the non-diff point(s). Classify them as standard (like $|x|$) or cusp(s) or VTL(s) (vertical tangent line). Justify briefly using a calculation of f' or f'', etc.

Bonus) Compute $\lim_{x\to 0^+} x^x \cdot (2x)^{2x} \cdot (3x)^{3x} \cdot (4x)^{4x}$

Remarks and Answers; The average was about 66, about like Exam 2. Use the same scale (please see E2 key). The best scores were on #2 (95% right!) and the worst were on #5 (35% right) - these were the two word problems.

I have averaged your 3 exam scores, and used that to estimate your semester grade so far (not including HW); see the upper right corner of your first page. The class average for this stat is about 63. So, the scale for that is about 3 points below the one for Exam 3. For example, the B's go from about 64 to 74, including $\pm$'s.

1a) -2, a fairly standard L'Hopital problem, Ch 3.6. Note; you can safely replace the $\cos(3x)$ term by $\cos(3\pi) = -1$ since it is continuous and nonzero (this simplifies the LHR work a bit).

1b) 1/3

1c) 0

1d) $e^1 = e$

1e) 0. This one is not indeterminant. You can use $0^\infty = 0$, though this notation is imperfect.

2) 70 by 35

3) Inc on $(-\infty, -2)$ and on $(3, +\infty)$. Concave up on $(1/2, +\infty)$. Etc. Use a graphing utility to check your graph. This was 4.2.53, and the results were good.

4) The only stat point is $\pi/3$ which gives the min. The max occurs at $-\pi/4$. Though the problem asks for the y values, I gave credit for these x values. This was 4.4.13; the results were a bit low.

5) -2 ft / sec. Few people got off to a good start, not labeling variables correctly. I drew the picture on the board, a triangle like the one in exercise 3.4.32. The base of it contains 2 important line segments. The first is the man's shadow and should be labeled (perhaps with s) because the question is about this variable. The second is the path from the man to the post, and should be labeled (perhaps with x), because the problem contains information about this ($dx/dt = -4$). No other variables, besides t, are necessary.

Try this again using x and s. It is not hard.

6) FTTTF

7) $x = 3$ is a nd point with a vertical tangent line. This is justified by computing f' and that $\lim_{x\to 3} f'(x) = +\infty$ (on both sides).

Bonus) $1^4 = 1$. Note $\lim_{x\to 0^+} x^x = 1$ (use the same method as 1d). Then $\lim_{x\to 0^+}(2x)^{2x} = 1$ too, by a substitution (or use the same method again). Etc.