MAC 2311 April 4, 2014
Exam III Prof. S. Hudson

1) Find equations for all vertical and/or horizontal asymptotes of $y = \frac{5x^2}{x^2-4}$.

2) Find the equation for the oblique asymptote of $y = \frac{5x^2}{x-3}$.

3) Analyze and graph $f(x) = x^3 - 3x + 1$. State clearly where it is increasing (etc). Plot and label any critical points and/or inflection points. I strongly suggest double-checking your work.

4) With the same $f(x)$ as in problem above, find the absolute extrema of f on the interval $[0, 2]$. Briefly explain your reasoning.

5) If $s(t) = t^3 - 3t + 1$ is the position of a particle at time $t \geq 0$, when is it speeding up? (note that s is almost the same as f above).

6) Compute these limits:

a) $\lim_{x \to \pi} \frac{\sin(2x)}{\pi - x}$

b) $\lim_{x \to 0} x \cot(x)$

c) $\lim_{x \to \infty} (1 - \frac{1}{2x})^x$

7) Mark each sentence True or False;

A graph can cross a horizontal asymptote.

If $0 < a < b$ then $\ln(x)$ has a maximum value on the interval $[a, b]$, and it is $\ln(b)$.

If $f''(c) = 0$ then f changes concavity at c.

A rational function can have 4 horizontal asymptotes.

If a polynomial f has 4 different roots, it must have at least 4 critical points.

8) An open box is to be made from a 16" by 30" piece of cardboard by cutting out equal squares from the four corners and bending up the sides. What size should the squares be to maximize the volume of the box?

9) Choose ONE proof:

a) Rolle's Thm
b) The MV Thm
c) If $f'(x) < 0$ on (a, b) then f is decreasing there.

Bonus: Give an example of a function $f(x)$ with a relative max at 1 and at -1, and a cusp at 0.

Remarks and Answers: The average was about 59, with highs of 77 and 73. The best results were on problems 1 and 3, the worst on 2 and 8. The scale is

A's 69 - 100
B's 59 - 68
C's 49 - 58
D's 39 - 48

The semester average is also about 59, still not counting HW, with a high of 82. You can use the same scale as above, though I have done this for you in the upper right corner of your exam.

1) HA: $y = 5$ VA's: $x = -2$ and $x = 2$.

2) $y = 5x + 15$ by using long division.

3) Your (long) answer should include phrases such as these:
It is increasing on $(-\infty, -1)$ and on $(1, \infty)$.
It is concave up on $(0, \infty)$.

Among other things, your graph should include the point (1,-1). It should be labeled (because it is a critical point) and it should look like a stationary point; the viewer should be able to imagine a horizontal tangent line there, and see that $f'(1) = 0$. Likewise, the concavity, etc, should match your calculations and statements.

4) Abs Max at (2,3). Abs Min at (1,-1). Do not include points like (-1,3) since $x \in [0, 2]$. Explain briefly that both extrema exist by the EVT, and that they must occur at endpoints or critical points in $(0, 2)$ (eg at 1).

5) It is speeding up when $t \geq 1$ (when v and a have the same sign). The grades were pretty good on average, but many answers were a bit off (eg $t = 1$ instead of $t \geq 1$ or $t \in [1, +\infty)$) and/or phrased badly. Often little or no work / reasoning was given. Note that the answer to such a question will usually be an *interval*, not a number.

6a) -2 from LHR.

6b) 1. One easy method is $\frac{x}{\tan x}$ followed by LHR.

6c) $e^{-1/2}$. Since the results were so low, here is the work:

Let A be the answer. So, $\ln(A) = \lim x \ln(1 - 1/2x) = \lim \frac{\ln(1-1/2x)}{1/x} = -1/2$ (using LHR and some easy algebra). So, $A = e^{-1/2}$. This problem can also be solved by the substitution $t = -2x$, without LHR.

7) TTFFF

8) Set $x = 10/3$. This is Ch. 4.5, Ex 2. The calculations near the end are a bit messy (I didn't notice this) and nobody got this completely right. But I gave about 5 points for

drawing the picture and setting it up;

$$V = HLW = x(16 - 2x)(30 - 2x)$$

Maximize this, with $0 \leq x \leq 8$.

9) See the text for these or similar proofs. Always aim to understand first, memorize as needed, practice, and explain with more words than you think you need. Understand first!