MAC 2311 Dec 8, 2011
Final Exam and Key Prof. S. Hudson

1) (25 pts) Compute and simplify;

$\int e^{x^2} x \; dx =$

$\int \frac{2}{1+x^2} \; dx =$

$\int \sin^2(t) \; dt =$

$\int \sec^2(t) \; dt =$

$\int \ln(e^{4t}) \; dt =$

2) (15 pts) Compute;

$\frac{d}{dx} x \sin(3x)$

$\frac{d}{dx} \cot(x)$

Find dy/dx, given that $x^2 + y^2 = 40$

3) (10 pts) Find the slope of the tangent line to the parametric curve, $x = \cos(t)$, $y = 3\sin(t)$ at the point where $t = \pi/3$. For maximum credit, use the chain rule as done in class.

4) (10 pts) Suppose a baseball player is moving at 25 ft/sec from 2nd base towards 3rd, when he is 20 ft away from 3rd. How fast is his distance from home plate changing at that instant ? Notes: the bases form a 90 ft square, and you can ask me for a picture. You do not have to simplify.

5) (10 pts) CHOOSE ONE;

A) State and prove Rolle's Theorem.

B) State the definition of limit, and use it to prove that $\lim_{x \to 5} 4x + 3 = 23$.

C) If $f'(x) > 0$ on (a, b) then f is increasing on (a, b).

6) (10 pts) Answer TRUE or FALSE:

If $f'(x) > g'(x)$ on $(1, 4)$, then $f(3) - f(2) > g(3) - g(2)$.

If f is increasing on (a, b) then $f'(x) > 0$ there.

A continuous function can have two different horizontal asymptotes.

The sum of any two continuous functions defined on $(-\infty, +\infty)$ is differentiable.

A continuous function defined on $[-3, 3]$ must have a minimum value.

7) (5pts) Let $f(x) = (x-3)e^x$. Find and the classify the (only) critical point, as a relative min, a relative max, or neither, *using the First Derivative Test.*

8) (5 pts) Solve the initial value problem; $y'(x) = (x+1)^3$ and $y(0) = 4$.

9) [10 pts] Suppose a particle has position $s(t) = t^3 - 6t^2 + 4$ and acceleration $a(t) = 6t - 12$ for $t \geq 0$. Find a formula for the velocity. When is the particle speeding up? slowing down? Explain briefly.

Remarks and Answers: Most of the papers were very good, with an average of about 75 / 100. The results were a bit low, but still over 50%, on Problems 5 and 6 (the proof and TF). I have not set a special scale for the final.

This brings the semester average up to about 67%, so I expect to use the original scale on the syllabus to assign letter grades (approx). I have not yet added in the HW and EC, but don't expect those to change the scale.

1a) $e^{x^2}/2 + C$

1b) $2\tan^{-1}(x) + C$

1c) $t/2 - \sin(2t)/4 + C$

1d) $\tan(x) + C$

1e) $2t^2 + C$

2a) $\sin(3x) + 3x\cos(3x)$

2b) $-\csc^2(x)$

2c) $-x/y$

3) $-\sqrt{3}$

4) $-500/\sqrt{8500}$

5) See text or lectures. Most people chose A (Rolle's).

6) TFTFT

7) $x = 2$ is a rel min.

8) $(x+1)^4/4 + 15/4$

9) Slowing down when $2 \leq t \leq 4$, and otherwise speeding up.