MAC 2311 April 23, 2014
Final Exam and Key Prof. S. Hudson

NAME

Show all your work and reasoning for maximum credit. Do not use a calculator, book, or any personal paper. You may ask about any ambiguous questions or for extra paper. Hand in any extra paper you use along with your exam.

1) [15 pts] For $f(x) = x^4 - 6x^2 + 5$

a) Find the intervals on which f is increasing; on which f is decreasing.

b) Find the critical points and determine whether a relative minimum, relative maximum or neither occurs there.

c) Find the intervals on which f is concave up; on which f is concave down.

d) Find the coordinates of all inflection points.

e) Graph the function.

2) [8 pts] Find $\frac{dy}{dx}$ where y is implicitly defined as a function of x by $x^3y^2 - 5x^2y + x = 1$.

3) [10 pts] A closed rectangular container with a square base is to have a volume of 2250 cubic inches. The material for the top and the bottom of the container will cost \$2 per square inch, and the material for the sides will cost \$3 per square inch. Find the dimensions of the container of least cost.

4) [8 pts] Evaluate $\lim_{x\to 0} \frac{1-\cos(3x)}{8x^2}$

5) [5 pts] Compute $\sum_{k=7}^{100} k$.

6) [8 pts] Sketch the curve by eliminating the parameter

$x = \sqrt{t}$

$y = 2t + 3$

7) [20 pts] Compute each, using any valid method. Be careful to answer with a number, a function, or a set of functions, as appropriate:

$\int \sin^2(x)\ dx$

$\int \tan(x)\ dx$

$\int \frac{1+t^3}{t^2}\ dt$

$\int_1^3 2x dx$

8) [8 pts] Use a local linear approximation to estimate $(1.97)^3$.

9) [8 pts] Find a value of the constant k that makes this function continuous everywhere

$$f(x) = \begin{cases} kx^2 & x \le 2 \\ 2x + k & x > 2 \end{cases}$$

10) [10 pts] Choose ONE: provide ample explanation, as usual.

a) State and prove the theorem about $\lim_{x\to 0} \frac{\sin(x)}{x}$.

b) Prove $\lim_{x\to 3} 2x = 6$ using the ϵ method.

c) Prove that $\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$, using the $(f^{-1})'(x)$ formula.

Bonus: [5 points] Compute the exact area under $y = x^2$ from $x = 2$ to $x = 4$ using integration (rectangles, summation formulas, a limit).

Remarks and Answers: The average was about 54 out of 100 with high scores of 81 and 78. The worst scores were on problem 3 (17%), 5 (35%) and 10 (40%), with problem 7 not much better. I will treat problem 5 more-or-less as extra credit since it comes from Ch. 5.4. I have not yet computed an average or set a scale for the semester grades.

1) [brief answer only] Increasing on $(-\sqrt{3}, 0)$ and on $(\sqrt{3}, \infty)$. The endpoints are the critical points, and could be included. It is concave down on $(-1, 1)$ and c.up elsewhere. The graph is shaped like a curvy W.

2) $\frac{dy}{dx} = \frac{10xy-3x^2y^2-1}{2x^3y-5x^2}$

3) Draw a picture and set $2250 = hx^2$, so $h = 2250x^{-2}$. You can include $V = lwh$ if it helps you, and you can use other letters for x (side-length of the base) and h (height). The cost is $C(x) = 4x^2 + 12xh = 4x^2 + 12 \cdot 2250 \cdot x^{-1}$, $0 \leq x < \infty$. Ch 4.4 methods (C'(x)=0, etc) give $x = 15$, $h = 10$.

By far the most common problem was setting this up, not getting $C(x) = 4x^2 + 12xh$, for example. The cost is the sum of the costs of the 6 faces; the bottom face (the base) will cost \$2 per square inch, times x^2 square inches, so $2x^2$ (etc). With practice, this should not be very hard.

4) 9/16 from LHR twice.

5) If the sum started at $k = 1$ the answer would be 5050 (the Little Gauss example). Since it starts at 7, some numbers (1+2+3+4+5+6=21) have been left out. So, 5050-21=5029.

6) $y = 2x^2 + 3$ with $x \geq 0$, half a parabola.

7) $x/2 - \sin(2x)/4 + C$, the main idea is $\sin^2(x) = (1 - \cos(2x))/2$ (done in class).
$\ln|\sec(x)| + C$, the main idea is $u = \cos(x)$ (done in class)
$-1/t + t^2/2 + C$, the main idea is $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$.
$x^2|_1^3 = 8$

8) $f(x_0) + f'(x_0)dx = 8 + 12 \cdot (-0.03) = 7.64$.

9) $k = 4/3$ Set $kx^2 = 2x + k$ with $x = 2$.

10) See the text or lecture notes; the common problem was failure to (re)study these proofs.

Bonus) 56/3 after a long calculation (see Ch.5.4). No credit for using $\frac{x^3}{3}|_2^4$.