MAC 2311 Dec 11, 2007

Final Exam and Key Prof. S. Hudson

1) (20 pts) Compute and simplify. Include an arbitrary constant, if appropriate;

$\int e^{2-3x}\ dx =$

$\int \frac{3}{\sqrt{1-x^2}}\ dx =$

$\int \frac{1-2t^3}{4t}\ dt =$

$\int \sin^2 t\ dt =$

2) (20 pts) Compute;

$\frac{d}{dx}\log_2(\sin(x))$

$\frac{d}{dx}\frac{\sqrt{4x}}{x+2}$

$\frac{d}{dx}\sec^{-1}(x+2)$

$\frac{d}{dx}x^{2x}$

3) (5pts) Suppose a particle has position $s(t) = t^3/3 - 2t^2 + 5$ [so, $v(t) = t^2 - 4t$ and $a(t) = 2t - 4$] for $t \geq 0$. When is the particle speeding up? slowing down? Explain briefly.

4) (10 pts) Compute;

a) $\lim_{x\to 0} \frac{\tan(3x)}{x\cos(4x)}$

b) $\lim_{x\to 0} \frac{2^x-1}{3^x-1}$

5) (10 pts) A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals (see the figure below or on the board). What dimensions should be used so that the enclosed area will be a maximum? [If you don't understand this story, ask me!]

The figure was a rectangle with a vertical line down the middle (so, two rectangles, side-by-side).

6) (10 pts) Given parametric equations, $x(t) = 2t^2+3$ and $y(t) = t^3+1$, find the equation of the tangent line at the point where $t = 2$, at $(x, y) = (11, 9)$.

7) (10 pts) Answer TRUE or FALSE:

The function $f(x) = |x - 2|^2$ is differentiable on $(-\infty, \infty)$.

A fourth degree polynomial must have a minimum value on $(-\infty, \infty)$.

The function $f(x) = \sec(4x)$, defined on $[0, \pi/6]$, has an inverse.

The function $\tan^{-1}(x^2)$ on $(-\infty, \infty)$ has an inverse.

If F is an antiderivative of an antiderivative of f, then $F\,''(x) = f(x)$.

8) (5pts) Suppose that $A(x)$ is the area under $y = \sqrt{x + 10}$, and directly above the line segment $[0, x]$. Find $A'(6)$, and explain briefly.

9) (10 pts) CHOOSE ONE;

A) State and prove Rolle's Theorem (as in class, you can omit the details of one case).

B) Prove that $\frac{d}{dx}\sin(x) = \cos(x)$, starting from the definition of derivative.

C) Prove that the derivative of a differentiable odd function is always an even function.

Remarks and Answers: The average score was about 65/100, which is normal (maybe better) for a final exam. But the average was only 48% on the True-False! The scores were good on problems 1 and 5; bad on 7 and 8.

1a) $-e^{2-3x}/3 \ + C$ (don't forget the +C ! I even reminded you!)

1b) $3\sin^{-1} x + C$

1c) $(\ln|x|)/4 - t^3/6 \ + C$

1d) $t/2 - \frac{\sin 2t}{4} \ + C$

2a) $\csc x \cos x / \ln 2$

2b) $2(x^{-1/2}(x+2)/2 - x^{1/2})/(x+2)^2$

2c) $[|x+2|\sqrt{(x+2)^2-1}]^{-1}$

2d) Log Diff; $y' = 2^{2x}[2\ln x + 2]$

3) Speeding up on $[0,2]$ and on $[4,\infty)$, where $av > 0$. Slowing down on $[2,4]$.

4a) 3

4b) $\ln 2/\ln 3$

5) $A = xy$ (x = width), $200 = 2x + 3y$, $y = (200-2x)/3$, $A = x(200-2x)/3 = (200x - 2x^2)/3$, $0 = A'(x) = (200-4x)/3$, so $x = 50$ and $y = 100/3$.

6) $y'/x' = 3t^2/4t|_{t=2} = 3/2$ so $(y-9) = (3/2)(x-11)$

7) TFFFT   Notes: $|x-2|^2 = (x-2)^2$; $f(x) = -x^4$; $\sec(4x)$ dne at $x = \pi/8 < \pi/6$; $\tan^{-1}(7^2) = \tan^{-1}((-7)^2)$, so not 1-1; and the last one is easy.

8) $\sqrt{10+6} = 4$ This should have been easy (see Ch 6.1, eqn 2; $A' = f$; and exercises 13-18).

9) see text/lectures