MAC 2311 AM April 24, 2009

Final Exam Prof. S. Hudson

1) (10 pts) Compute y';

a) $y = (2x)^x$

b) $y = \log_3(2x + 1)$

2) (10 pts) Solve this Initial Value Problem: $y'(t) = e^t + t$ and $y(0) = 2$.

3) (25 pts) Compute and simplify;

$\int e^{2x}\ dx =$

$\int \frac{t}{1+16t^2}\ dt =$

$\int \frac{1-2t^3}{t^3}\ dt =$

$\int \cos^2(x)\ dx =$

$\int \tan^2(x)\ dx =$

4a) (10 pts) Find the slope of the tangent line to the curve, $x = \sqrt{t}$, $y = 2t+1$ at $t = 1$. For maximum credit, use the chain rule as done in class.

4b) (5 pts) For the same curve as above, find d^2y/dx^2 when $t = 1$.

5) (10 pts) Sketch a graph of $y = \frac{x^2-1}{x^3}$. Find all critical points, inflection points and asymptotes [and label them clearly]. You may use:

$y' = \frac{3-x^2}{x^4}$
$y'' = \frac{2(x^2-6)}{x^5}$
$\sqrt{3} \approx 1.8$
$\sqrt{6} \approx 2.4$
$1.8^{-3} \approx 0.18$
$2.4^{-3} \approx 0.07$

6) (20 pts) Answer TRUE or FALSE:

$f(x) = \ln|x|$ is an increasing function.

A rational function is continuous except where the denominator is zero.

The function $\cot(x)$ is continuous on the interval $(-\pi/4, \pi/4)$.

If f is a polynomial, then it has exactly one antiderivative whose graph contains the origin.

If $f(2) = f(3)$ then there is a point $c \in (2,3)$ where $f'(c) = 0$.

If F is an antiderivative of an antiderivative of f, then $F''(x) = f(x)$.

$f(x) = x^{2/3}$ is a differentiable function.

$f(x) = (2x^2 + 1)/x$ has an oblique asymptote of $y = 2x$.

The graph of $r = \sin(\theta)$ is a circle.

Both ∞^0 and 0^∞ are indeterminate forms.

7) [10pts] Suppose a particle has position $s(t) = t^3/3 - 2t^2 + 5$ [so, $v(t) = t^2 - 4t$ and $a(t) = 2t - 4$] for $t \geq 0$. When is the particle speeding up? slowing down? Explain briefly.

Remarks: The average was about 65 /100 based on 17 scores over 40. The high score was 96. The only unusual statistics were an 83% average on 7), and a 56% average on the TF. I thought a few parts of problem 3 were relatively difficult (for Calc I) and was happy to see good results there. You can use the standard scale (on the syllabus) for this exam, or maybe lower it a few points in your favor.

1a) $(2x)^x[\ln|2x| + 1]$

1b) $2/(\ln(3)(2x+1))$

2) $y(t) = e^t + t^2/2 + 1$

3a) $e^{2x}/2 + C$

3b) $\ln(1 + 16t^2)/32 + C$

3c) $-t^{-2}/2 - 2t + C$

3d) $x/2 + \sin(2x)/4 + C$

3e) $\tan(x) - x + C$

4) 4 and 4

5) This is example 2 on page 292 (with some of the work already done).

6) FTFTF TFTTF

7) Speeding up on $(0, 2)$ and $(4, +\infty)$