MAC 2311 Jan 18, 2006
Exam on Pre-Reqs, and Key Prof. S. Hudson

1) What is an equation of a line through (-4,7) and perpendicular to $y = \frac{2}{3}x + 5$?

2) Solve for x, given that $\log_2 x^3 = 6$ (and simplify).

3) Solve for x, given that $\sin(x/2) = 1/2$.

4) Solve for x, given that $\frac{1}{3-9x} < \frac{1}{11}$; answer with interval(s).

5) Simplify the formula $3x + \sin^2(2x) + \tan^2(3x) + \cos^2(2x) - \sec^2(3x)$.

6) Suppose $x_1 = 1$, $x_2 = 4$, $x_3 = 7$ and $x_4 = 10$ (etc). Find a formula for x_k.

7) Suppose that x is a number so that $|x - 1| < 2$. Explain in detail why $|x - 2| < 3$ must also be true. Your answer can be mostly formulas, but it should include some words, too.

Remarks: I wanted the questions to reflect a wide variety of skills you will need in Calc I-II, so I chose them from several sources: Prof De Carli's review page (#3 and #4), the textbook, small steps needed in my old Calculus exams, and #1 was from my daughter's geometry book. Problems 1, 2, 3, and 5 were relatively easy (with average scores 60-80 per cent among the best 20 students).

If you scored **70-100**, you are ready for Calculus.
If you scored **50-70**, you may pass Calc I, probably with some difficulty.
If you scored **0-50**, you probably will not pass Calc I with a C.
The previous 3 lines are just advisory, based on statistics from prior classes. But -

If you scored **0-40, then you must see me**, about additional testing or pre-calculus practice, if you want to continue in this class. I will not "drop" you from the class roll - you should do that yourself. If you simply disappear, you will get an F for the semester.

Answers: But ... as in most answer keys, I don't always show all the required work. You can see me about any of these that you don't understand.

1) The new slope will be the negative reciprocal of the old one, so -3/2. The equation is $y - 7 = -3/2(x - (-4))$ which can be simplified to $y = -3x/2 + 1$.

Suggestion: check your answer by plugging in $x = -4$ to get $y = 7$. I do not require checking, but I give less partial credit if an answer is *obviously* wrong.

2) Change to $3 \log_2 x = 6$, then $\log_2 x = 2$, then $x = 2^2 = 4$. [Or, you can do $x = (2^6)^{1/3} = 4$, but I think there were a few more mistakes that way].

3) Since $\sin(\pi/6) = 1/2$, we can set $x/2 = \pi/6$ and get $x = \pi/3$. I gave full credit for this, but I was too generous, because of other solutions, like $-8000\pi + 5\pi/3$. Better:

$x \in \{\pi/3 \pm 4n\pi\}$ or $x \in \{5\pi/3 \pm 4n\pi\}$ where $n \in \{0, 1, 2, 3 \ldots\}$

4) **Method A**: Assume that $3-9x > 0$ so that we can cross-multiply, and get $11 < 3-9x$. Which means $9x < -8$ and $x < -8/9$. All these x's will be in the solution set, since they also make $x < 1/3$ true, so that $3-9x > 0$ is also true.

Now assume that $3-9x > 0$ is not true, so that $3-9x < 0$, which is the same as $x > 1/3$ (the 2 sides cannot be equal, because that creates a zero in a denominator). These x's will make the original inequality true, because the left side will be negative.

Final Answer: $x \in (-\infty, -8/9) \cup (1/3, \infty)$. Other notation was acceptable.

Method B: If you get lost in the logic of Method A, just calculate the important numbers first. You get the 1/3 by setting the denominator equal to zero. You get the -8/9 by setting $\frac{1}{3-9x} = \frac{1}{11}$. These split the number line into three intervals: $I_1 = (-\infty, -8/9)$ and $I_2 = (-8/9, 1/3)$ and $I_3 = (1/3, \infty)$ and the final answer will be made from some of these. We can check that numbers in I_1 make the original inequality true by testing any one of them (eg check that $x = -100$ makes it true). Likewise, it is easy to check that numbers in I_3 are OK, but ones in I_2 are not. So, we get to the same answer as in Method A.

Think about why Method B works. We will use it later in Calc I. In fact, to encourage you a little on this, I may ask another question like problem 4 on Exam I.

5) After using 2 trig identities, you get $3x + 1 - 1 = 3x$.

6) $x_k = 3k - 2$

7) First, I might have to solve the inequalities or draw a number line (to get the right idea). But then I'd write something like this:

Assume that $|x-1| < 2$.
So, $-2 < x - 1 < +2$ (by an algebra rule).
Solving for x by adding 1 to each part; $-1 < x < +3$.
By subtracting 2, $-3 < x - 2 < 1$.
Since $1 < 3$, the transitive property implies $-3 < x - 2 < 3$.
So, $|x-2| < 3$ (same algebra rule). Done.

You could combine lines 3 and 4 by subtracting 1 from each part of $-2 < x - 1 < +2$.

There are other ways to explain this. You could solve both inequalities, and say something like this: "The solution set of the first inequality is [-1,3], which is contained in [-1,5], which is the solution set of the second inequality. So, any solution of the first one also makes the second one true."

But you must not *assume* that $|x-2| < 3$.

The formula $-2 < x - 1 < +2$ can be restated as $x - 1 < 2$ *and* $-2 < x - 1$. But do not replace the word *and* by *or*. These little words are very important in explaining the logic of a proof. They will affect your grade as much as minor calculation errors.