MAC 2311 Jan 16, 2007
Quiz on Pre-Reqs Prof. S. Hudson

Problems 2-5 are worth 15 points each.

1) [10pts each] In each part, solve for x. In some parts, the answer might be a single number. In others, it might be several numbers, or an interval, etc.

a) $\frac{1}{x-3} + \frac{1}{x+3} = \frac{4}{x^2-9}$

b) $(x-5)(x+5)/(x-10) < 0$

c) $\log_3(3^x) = x^{1/2}$

d) $\sin(x) = \sin(2x)$ with $\pi \le x \le 2\pi$.

2) Solve for both x and y:

$$\begin{aligned} x - 2y &= 2 \\ -2x + 8y &= -2 \end{aligned}$$

3) Simplify $\sin(\cos^{-1}(x))$ to an algebraic expression.

4) Which of these are 1-1 functions of x ? Circle the ones that are.

$|x| \qquad x^3 \qquad \sin(x) \qquad \tan^{-1}(x) \qquad e^x$

5) Suppose that x is a number so that $|x-3| < 2$. Explain in detail why $|x-4| < 3$ must also be true. Your answer can be mostly formulas, but it should include some words, too.

Remarks and Answers: The average was about 45/100. If you scored below 40/100, you probably do not have the background you need to pass Calculus I, and *you must see me* about that. If you scored in the 40-50 range, your prospects are not too good, and you *may* also want to see me about remedial work. Assuming that most low scorers will drop, the average will probably rise to about 60/100. The approx scale is:

A's 75-100, B's 65-74, C's 55-64, D's 45-54, F's 00-44.

The questions were mostly taken from Ch 1 exercises, from standard PreCalc exams, or from my Spring 2006 pretest (with some minor changes). The average scores were high on 1a and 2; low on 1b, 1c, 1d.

1a) $\frac{1}{x-3} + \frac{1}{x+3} = \frac{(x+3)+(x-3)}{x^2-9}$, so $2x = 4$. So, $x = 2$.

b) Let $f(x) = (x-5)(x+5)/(x-10)$. This function changes sign at -5, 5 and 10. For example, $f(4.99) > 0$ but $f(5.01) < 0$. You don't need a calculator to see that - just look at the signs of the three factors separately. After checking another point or two (such as $f(-6) < 0$ and $f(11) > 0$), we can be sure the solution set is $(-\infty, -5) \cup (5, 10)$.

I usually like to organize this kind of work on a number line, but it's hard to type.

c) $\log_3(3^x) = x$, so $x = x^{1/2}$. We can square both sides to get $x^2 = x$ and $x^2 - x = 0$ and $x(x-1) = 0$. So, $x = 0$ or $x = 1$.

Note: Squaring both sides can "create new solutions", so we really should check both 0 and 1 at the end. But they both satisfy $x = x^{1/2}$ and are both OK.

Several people answered $x = 1$ (without doing much work, and without finding $x = 0$). I had to assume that was guesswork, and didn't give a lot of partial credit for it.

d) $\sin(x) = \sin(2x)$ with $\pi \le x \le 2\pi$. Since $\sin(2x) = 2\sin(x)\cos(x)$, the equation soon becomes $\sin(x)[1 - 2\cos(x)] = 0$. So, either $\sin(x) = 0$ or $1 - 2\cos(x) = 0$. We know $\cos(\pi/3) = 1/2$, but $\pi/3$ is not in the given interval. The answer is: $x = \pi$ or 2π or $2\pi - \pi/3 = 5\pi/3$.

2) $x = 3$ and $y = 1/2$. Multiply the top eqn by 2 and add that to the second one, etc. [It is OK to solve one eqn for x, and plug the result into the second eqn, but that method is probably more work, especially for bigger versions of this problem].

3) $\sin(\cos^{-1}(x)) = \sqrt{1-x^2}$.

Set $\cos^{-1}(x) = \theta$, so $\cos(\theta) = x$. To find $\sin(\theta)$, I'd draw a triangle as done in class, but it is also OK to start from $\sin^2(\theta) + \cos^2(\theta) = 1$, and get $\sin(\theta) = \pm\sqrt{1 - \cos^2(\theta)} = \pm\sqrt{1-x^2}$. Then explain that the $\pm$ is $+$, because θ is in the range of $\cos^{-1}(x)$, which is $[0, \pi]$. But I didn't insist on so much explanation, with this method or the triangle one.

4) These are 1-1: x^3, $\tan^{-1}(x)$ and e^x. Each one has a well-known inverse function, and each passes the horizontal line test. The other two fail it. You should already have a pretty good mental image of the graphs of these 5, but you could plot some points if needed.

5) This is very similar to a problem on the pre-test in Spring 2006. So, please review that answer key. Another quick approach is to calculate and compare the two solution sets (and explain);

$$(1, 5) \subset (1, 7)$$

But I wanted to make you think and write (rather than just calculate), so I'd prefer the method on the 2006 key.