MAC 2312 | Sept 16, 2014
Exam I | Prof. S. Hudson

1) (5pts) Compute the sum, using a summation formula if possible. Simplify, within reason (fractions are OK, but do not leave a $\sum$ symbol, or many + signs in your answers).

$\sum_{k=1}^{10} k(k-2)$

2) (10pts) Compute this limit by interpreting it as a definite integral (and then compute that integral). Of course, you can put the $2/n$ term inside the sum if you want.

$$\lim_{n\to\infty} \frac{2}{n} \sum_{k=1}^{n} \left(\frac{2k}{n}+1\right)^9 =$$

3) (5pts) Let $F(x) = \int_2^x \log_3(t)\ dt$. Compute $F'(9)$ and simplify.

4) (7pts each; 35 total) Compute each integral (by substitution, geometry, FTC, inv. trig, etc). In most cases, you should simplify your answer. If any of these do not exist, explain.

$\int_0^{\pi/4} \sec^2 x\ dx$

$\int_0^1 \sqrt{1-t^2}\ dt$

$\int_0^{1/3} \frac{dx}{1-x}$

$\int_0^{\pi/4} \sin^3(x)\cos(x)\ dx$

$\int_0^4 |1-\sqrt{x}|\ dx$

5) (10pts) Find the displacement and distance traveled by an object with velocity $v(t) = \cos(t)$, in feet per second, with $\pi/2 \le t \le 2\pi$.

6) (10pts total) a) Find the average value of $f(x) = \frac{1}{x}$ on $[1, e]$.

b) Find the value of c [in the MVT] so that $f(c) =$ the average value from part (a).

7) (15pts) Answer True or False to each part. You do not have to explain.

The function $f(x) = \sec(x)$ is integrable on $[0, \pi/4]$.

If f is continuous on (1,4), then it is integrable on [1,4].

If $a < b$ and f is continuous on [a,b], then $\int_a^b f(x)\ dx > 0$.

Using the Left Endpoint Rule on $\int_1^5 x^3\ dx$ will produce a Riemann Sum less than the exact value.

The sum $1 + 3^{-1} + 3^{-2} + \ldots 3^{-n}$ is always less than 2, for all $n > 0$.

8) (10pts) Choose ONE proof. If possible, use sentences or formulas with complete justifications. Recall that the grading will be based on the clarity of your logic and explanations, as much as on any calculations involved.

A. State and prove the FTC #2 (about $\frac{d}{dx}\int$).

B. State and prove the formula for $\ln(ab)$.

Bonus: (5 points, maybe hard): Calvin and Hobbes each compute a Riemann sum for $\int_0^{\pi} \sin(x)\ dx$, both using $n = 4$. Calvin gets 3 and Hobbes gets 4. Can both of these be correct ? Explain.

Remarks, Scale and Answers: The average among the top half was approx 61 out of 100, which is a bit low. The highest scores were 83 and 81. The average grade was about the same on all the problems except for a 15% average on problem 2, and approx 0 on the bonus. I went over # 2 in class but probably will not go over the others. You should review the answers below until all the problems make sense; you can ask me (in office hours) or your LA for help with that. Here is the adjusted scale for the exam, replacing the one on your syllabus (these are all just estimates until the end of the term):

A's 73-100
B's 63-72
C's 53-62
D's 43-52

1) 275 (I also accepted 825/3, etc). Use $k^2 - 2k$ and $n(n+1)(2n+1)/6$ and $-2n(n+1)/2$, both with $n = 10$.

2) $\int_1^3 x^9\ dx = \frac{3^{10}-1}{10}$. This problem is probably difficult without practice (moral? do your HW!). One way to start is to set $1 + 2k/n = x_k = a + k\Delta x$, so $a = 1$ and $2/n = \Delta x = (b-a)/n$. So, $b = 3$. Also, $f(x_k) = x_k^9$, and we get to $\int_1^3 x^9\ dx$.

It is possible to view the problem differently and get $\int_0^2 (x+1)^9\ dx$ instead, but it leads to the same final answer.

3) Using the FTC, get $\log_3(9) = 2$. I gave small partial credit for using $\log_3(t) = \ln(t)/\ln(3)$ (this formula is not very useful here, but it often helps in similar problems). I took off a point or so for stopping at $\log_3(9)$, because the simplification is quick, not tedious (and see the instructions).

4a) $\tan(\pi/4) - \tan(0) = 1$. Again, this should be simplified, though that was not a big part of the grade.

4b) $\pi/4$ (draw a picture, it is one fourth of a circle).

4c) $-\ln(2/3) = \ln(3/2) = \ln(3) - \ln(2)$. Any one of these is OK - I don't regard any as being much simpler than the others. But do use $\ln(1) = 0$ to simplify.

4d) After $u = \sin(x)$ get $\frac{u^4}{4}\Big|_0^{\sqrt{1/2}} = 1/16$.

4e) 2. Apply the *abc* theorem, splitting $[0, 4]$ into $[0, 1]$ and $[1, 4]$, getting $1/3 + 5/3 = 2$.

5) Displacement $= -1$. Distance traveled $= 3$, from applying the *abc* theorem (as in 4e, this is standard with absolute values) to $\int_{\pi/2}^{2\pi} |\cos(x)|\ dx = 3$. You might use a graph instead, but if so, explain your reasoning. Also, be sure to label your answers clearly, especially when a problem like this contains two questions.

6a) $1/(e-1)$. 6b) $e-1$. It is worth checking that $e-1 \in [1, e]$, as stated in the MVT, but I did not require that.

7) TFFTT

8) See the lectures or the text. The most common problem was a lack of explanation (see the instructions for this problem). For example, many answers looked like $\ln(ab) = \int_1^{ab} \frac{dt}{t} = \ldots = \ln(a) + \ln(b)$, with no words included. Ideally, you should mention (not just use) *the definition of* $\ln(x)$ (three times!) and *the abc thm* and *a u-substitution.*

There is some flexibility in how you explain a proof, but I feel these require at least 3 sentences each (perhaps depending on your writing efficiency), in addition to formulas. Face-to-face, we may take some shortcuts, but on an exam try to write everything a skeptical reader might need.

Bonus: The largest possible R.Sum for this is π (because $|f(x_k^*)| \le 1$). So Calvin's answer may be right, but Hobbes's cannot be.

Many people talked about whether the two answers should be equal. They do not have to be, but that idea is not the key to this problem.