MAC 2312 | Sept 19, 2016
Exam I Key | Prof. S. Hudson

1) (5pts each) Compute each sum.

$\sum_{k=1}^{10} k(k^2+1)$

$\sum_{k=10}^{20} 2^k$

$4+8+12+\cdots+2012+2016=?$ Convert to Σ notation, then use a summation formula.

2) (5pts each) Compute each part, using methods from Ch.5 (or earlier):

$\int_0^1 (10^x)^2\ dx$

$\int_0^4 \sqrt{16-t^2}\ dt$

$\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\ dx$

$\frac{d}{dx}\int_2^{e^x} \ln(t)\ dt$

$\int_0^{\pi} \sin^2(y)\ dy$

The next five are 5pts each. Remember to show all work and explain as needed.

3) Suppose a particle moves with velocity $v(t)=\sin(t)$ for $0 \le t \le 3\pi/2$. Find the distance traveled.

4) Let R be the region bounded by $y=0$, $y=1+\cos(x)$, $x=0$ and $x=\pi$. Sketch R. Find the volume of the solid generated when R is revolved around the $x-$axis, set-up only (find the integral that gives the answer, but do not compute the integral). State which 'Method' you are using.

5) Suppose you are working on the Riemann sum for $\int_2^5 x^2+1\ dx$ with $n=6$ and the LER (the left endpoint rule, with a regular partition). Find x_3^* and $f(x_3^*)$ in that context. You do not have to compute the whole Riemann sum or the integral.

6) State the definition of the definite integral $\int_a^b f(x)\ dx$. This should include a lim and a Σ and (for maximum credit) some remarks on the notation used and whether the f is integrable.

7) Solve for x, given that $\int_1^{x^2} \frac{dt}{t} = 6$.

8) (10 pts) Compute the area under $f(x)=x^2+5\ dx$ for $1 \le x \le 3$ exactly, using Riemann sums and a limit. Do not use an antiderivative, except maybe to check your answer. Since this is a rather long problem, here is some help getting started, using the RER. You can

ignore the help (if you do not understand my notation, for example), or you can start from here:

$A_n = \frac{2}{n} \sum_{k=1}^{n} f(x_k)$ where $x_k = 1 + \frac{2k}{n}$.

9) (15pts) Answer True or False to each part. You do not have to explain.

If f is continuous on $[0, \pi]$, then is has an antiderivative there.

If f is continuous on $[0, \pi]$, then is integrable there.

The function $f(x) = \tan(x)$ is integrable on $[0, \pi]$.

If velocity is positive for $0 \leq t \leq 5$ seconds, then the distance traveled is the displacement.

If $F'(x) = x^2$ on $[0, 5]$, then $F(x) = x^3/3$ there.

10) (10pts) Choose ONE proof. If possible, use sentences or formulas with complete justifications. Recall that the grading will be based on the clarity of your logic and explanations, as much as on any calculations involved.

A. State and prove the FTC #2 (about $\frac{d}{dx} \int$).

B. State and prove the formula for $\sum_{k=1}^{n} k$.

Bonus: (5 points) Try this if you have extra time, but it may be hard. Find a function $f(x)$ that makes the following equation true for all x, or explain why it is impossible. Hint: you could start by taking a derivative of both sides.

$$\int_1^{x^2} 2te^{t^4} + 1 \; dt = f(x) + \int_1^{x^4} e^{t^2} \; dt$$

Remarks and Answers: The average score was 58% among the top 20 scores, with high scores of 89 and 81. This is a bit low, so I have adjusted the advisory scale, see below. The results were good on problems 1 and 4, both a bit over 75%. They were low on problems 2a and 3 (about 20%) and on 6 (45%). Compared to most of my past exams, this one contained many short 5 point problems, which may have resulted in less partial credit overall.

A's - 67 to 100
B's - 57 to 66
C's - 47 to 56
D's - 37 to 46

1a) $\sum_{k=1}^{10} k^3 + k = 55^2 + 55$ with no need to simplify further.

1b) $\sum_{k=1}^{20} 2^k - \sum_{k=1}^{9} 2^k = \cdots = 2^{21} - 2^{10}$, geometric. In this case, you should do the easy simplifications behind this answer (such as 1-2=-1, etc).

1c) $\sum_{k=1}^{504} 4k = 4(504)(505)/2$. You should probably simplify this a bit, though I gave full credit for this answer (this time).

2a) $99/\ln(100)$. The results on this one were pretty bad. The main idea (from both Calc I and Ch.5.10) is to convert 'difficult' bases like 10 into base e. Start with $(10^x)^2 = 100^x = e^{(\ln(100))x}$ and then integrate with a u-subn, the same way you would handle $\int e^{5x}dx$.

Problems with $\log_{10} x$ (though not appearing on this exam) should also be converted (to $\ln(x)$ formulas) before doing calculus to them.

2b) 4π. This is easy if you graph $f(x)$, a quarter-circle.

2c) $\sin^{-1}(x)|_0^{1/2} = \pi/6$. This is mostly a Calc I memorization problem. Review more as needed. The results were rather low on average.

2d) xe^x. This is from the Chain Rule and FTC2, $f'(g)g'(x)$ with f being the integration and $g = e^x$. So, $f'(g) = \ln(e^x) = x$ and $g'(x) = e^x$.

2e) $\pi/2$. Replace $\sin^2(y)$ by $(1 - \cos(2y))/2$ and proceed normally ($u = 2y$, etc).

3) 3. By definition, DT is $= \int_0^{3\pi/2} |v| = \int_0^{\pi} v + \int_{\pi}^{3\pi/2} [-v] = 2 + 1 = 3$ (using abc to deal with the abs vals). With practice, you can probably get this faster and easier from a graph of v, by adding the areas above and below the x-axis.

If you got $-\cos(3\pi/2) + \cos(0) = 1$, then you probably computed displacement by mistake.

4) $V = \int_0^{\pi} \pi(1 + \cos(x))^2 \, dx$ using the Disk Method.

5) $x_3^* = x_2 = 3$. This should be easy, either from a number line picture or from the usual formula with Δx. Also, $f(x_3^*) = 3^2 + 1 = 10$. For some reason there were many silly mistakes on this one.

6) See the textbook. Most of the answers got at least some partial credit for including lim and $\sum$ in a reasonable way, but various minor mistakes were also common; $n \to \infty$ and Δx are not quite right (since irregular partitions are allowed). You should include the rule that $x_k^* \in [x_{k-1}, x_k]$ and discuss briefly whether the limit exists. It is true that continuous functions are integrable, but that is a theorem, not part of the definition.

7) $x = \pm e^3$.

8) Using the hints, and definition of f, $A_n = \frac{2}{n} \sum_{k=1}^{n} (1 + \frac{2k}{n})^2 + 5$. Then using algebra and summation formulas, $A_n = \frac{2}{n}[6n + 2(n+1) + \frac{4(n+1)(2n+1)}{6n})]$ The answer is $\lim_{n\to\infty} A_n = 12 + 4 + 8/3 = 56/3$.

9) TTFTF

10) See the textbook.

Bo) Setting $x = 1$ on both sides shows that $f(1) = 0$. Derivatives and FTC#2 lead to $4x^3e^{x^8} + 2x = f'(x) + 4x^3e^{x^8}$, so $2x = f'(x)$ and $f(x) = x^2 - 1$. Two people came very close, answering with $f(x) = x^2 + C$.