MAC 2312 Feb 2, 2016
Exam I and Key Prof. S. Hudson

1) (Short answer: 7pts each)

$\sum_{k=10}^{40} k$

$\frac{d}{dx} \int_7^x \sec(t)\ dt$

$\int_1^2 \frac{d}{dx} x^x\ dx$

Find the average value of $f(x) = \sin^2(x)$ on $[0, \pi]$.

2) (7pts each) Compute each integral (by substitution, geometry, FTC, inv. trig, etc). In most cases, you should simplify your answer. If any of these do not exist, explain.

$\int_0^{\pi/4} \sec^2 x\ dx$

$\int_0^2 \sqrt{4 - t^2}\ dt$

$\int_0^4 |1 - x|\ dx$

$\int_1^{1/3} \frac{dx}{x}$

$\int_0^{\pi/4} \sin(x)\cos^2(x)\ dx$

3) (15pts) Answer True or False to each part. You do not have to explain.

The distance traveled by a particle is always nonnegative.
Using the Right Endpoint Rule on $\int_1^5 x^3\ dx$ will produce a Riemann Sum less than the exact value.
The sum $1 + 2^{-1} + 2^{-2} + \ldots 2^{-n}$ is always less than 3, for all $n > 0$.
The average value of $f(x) = x^2$ on $[1, 5]$ is equal to $f(3)$.
The average value of $f(x) = 20x + 7$ on $[1, 5]$ is equal to $f(3)$.

4) (12 pts) Find the value of (a) $\sum_{k=1}^{n} f(x_k^*)\Delta x_k$ and (b) max Δx_k, given that
$f(x) = 4 - x^2$, $a = -3$, $b = 4$, $n = 4$.
$\Delta x_1 = 1$, $\Delta x_2 = 2$, $\Delta x_3 = 1$, $\Delta x_4 = 3$.
$x_1^* = -5/2$, $x_2^* = -1$, $x_3^* = 1/4$, $x_4^* = 3$.

Don't forget to do part 4b and circle or label your answers.

5) (10pts) Choose ONE proof. If possible, use sentences or formulas with complete justifications. Recall that the grading will be based on the clarity of your logic and explanations, as much as on any calculations involved. You can use the back of this page, but leave a note here.

A. State and prove the FTC #2 (about $\frac{d}{dx}\int$).

B. State and prove the formula for $S = \sum_{k=0}^{n} ar^k$.

Bonus: (5 points, maybe hard): Calvin and Hobbes each compute a Riemann sum for $\int_0^\pi \sin(x)\ dx$, both using $n = 4$. Calvin gets 0.5 and Hobbes gets 3. Can both of these be correct ? Explain.

Remarks and Answers: The average score was 76, based on the top 22 scores, which is very good. The high score was 103, followed by 3 scores in the low 90's. The average scores were similar on every problem, but were a bit higher on problem 2 and lower on 4. Here is a fairly realistic [and unofficial] scale for the exam, based on the high average. Eventually, the scale for the semester will come down to the one on the syllabus, or lower.

A's 84 to 100
B's 74 to 83
C's 64 to 73
D's 54 to 63

1a) 775. There are several good methods, such as $\sum_{k=1}^{40} k - \sum_{k=1}^{9} k = \frac{n(n+1)}{2}\Big|_9^{40} = 820 - 45 = 775$. Common mistakes: Using a 10 instead of a 9, or not subtracting off the second summation at all.

1b) $\sec(x)$ by FTC#2. Common mistakes: including $\pm\sec(7)$ or $+C$ or trying to find an antiderivative of $\sec(t)$ (it is possible, but rather hard, and not necessary).

1c) $x^x|_1^2 = 3$ by FTC #1.

1d) $\frac{1}{\pi}\int_0^\pi \sin^2(x)\ dx = 1/2$. This uses a double-angle trig id (know it!) and that the $\cos(2x)$ term cancels itself out.

2a) $\tan(x)|_0^{\pi/4} = 1 - 0 = 1$.

2b) area of part of a circle, $\frac{1}{4}\pi r^2|_{r=2} = \pi$. Though it may be possible to find an antiderivative later (Ch.7), that is a strategic mistake because it is harder.

2c) 5. The standard method using the abc thm and the definition of absolute value is $\int_0^1 1 - x\ dx + \int_1^4 x - 1\ dx = 1/2 + 9/2 = 5$. But in this example, it seems easier to graph the function and add the areas of the 2 triangles, $1/2 + 9/2 = 5$. Most people had the right idea, and most mistakes were minor ones.

2d) $\ln(1/3)$ [or $= -\ln(3)$] based on the definition of ln or from the FTC. Simplification such as $\ln(1) = 0$ counted approx 1 point. See the instructions for (2), but this kind of simplification is usually required anyway.

2e) $-u^3/3 \,|_1^{1/\sqrt{2}} = [1-2^{-3/2}]/3$ coming from $u = \cos(x)$. Simplifications such as $\cos(0) = 1$ counted approx 1 point each.

3) TFTFT. The results were good except for (3e). The midpoint rule is exact for linear functions (try it with n=1 and notice that the area of the trapezoid equals the area of the rectangle, based on simple geometry - or just ask).

4a) $1(4-(-5/2)^2) + 2(4-(-1)^2) + 1(4-(1/4)^2) + 3(4-(3)^2) = -9/4 + 6 + 63/16 - 15$ [which simplifies to $-117/16$ but I did not grade that].

4b) 3. This was an assigned HW problem.

5) Part A is in the book and lectures and Part B was HW. Most people chose A and did a pretty good job of including all the steps and justifications. Most mistakes were minor. You should state the theorem before proving it, and should assume f is continuous before stating the main formula, but these are fairly minor points.

B) Yes. The ideal justification would be create two specific RSums, equal to 0.5 and 3, but that might get messy. I gave full credit if you explained that the minimum possible sum is zero and the maximum one is π. So both 0.5 and 3 do seem possible. This logic probably depends [but I have not checked this carefully] on using irregular partitions, so I gave partial credit for mentioning those.