MAC 2312 2pm Feb 1, 2018
Exam I Key Prof. S. Hudson

Short answer: 7pts each. If any of these do not exist, explain.

1) $\sum_{j=1}^{20} \ j(j+2)$

2) $\sum_{k=12}^{20} k^2$

3) $\int_0^1 2^x \ dx$

4) $\int_0^\pi \sin^2(x) \ dx$

5) $\int_{-1}^2 \sqrt{2+|x|} \ dx$

6) $\frac{d}{dx} \int_2^{e^x} \ln(t) \ dt$

7) $\int_0^1 \frac{dx}{1+x^2}$

8) [10pts] A particle moves with velocity $v(t) = e^{-t}$ for $0 \le t \le 4$. Find the average velocity of the particle during this period.

9) [10pts] Compute the area under $y = x^2$ from $x = 1$ to $x = 2$ using Riemann sums and integration (not an antiderivative). Use the right endpoint rule with regular partitions. Use summation formula(s) and a limit. If you continue on the back, leave a note (as usual).

10) [8 pts] A car stopped at a toll booth leaves at time $t = 0$ with a constant acceleration of $a = 4$ ft/sec^2. How long will it take for the car to go 800 feet ?

11) [15pts] Answer True or False to each part. You do not have to explain.

The average value of $f(x) = x^4$ on $[1, 7]$ is equal to $f(4)$.

If f is continuous on $[a, b]$, then is has an antiderivative there.

If f is integrable on $[-5, 0]$, then it is also differentiable there.

The area between the curves $y = 2x$ and $y = x^2$ is $A = \int_0^2 2x - x^2 \ dx$.

If velocity is positive for some time interval, then the distance traveled is the displacement.

12) [8 pts] Choose ONE proof. Use sentences and formulas with complete justifications. The grading will be based on the clarity of your logic and explanations, as much as on any calculations involved. If you use the back of the page, leave a note here.

A. State and prove the FTC #2 (about $\frac{d}{dx} \int^x$).

B. State and prove the formula for $\ln(ab)$.

Bonus: (5 points, maybe hard): Calvin and Hobbes each compute a Riemann sum for $\int_0^{2\pi} \sin(x) \ dx$, both using $n = 4$. Calvin gets -5.0 and Hobbes gets 5.0. Can both of these be correct ? Explain.

Remarks and Answers: The average among the top 25 was about 70, which is fairly good. The two highest scores were 96 and 92. The grades were similar on all the problems, from about 60% on 5-9 to about 80% on 10 and 11. The unofficial scale for this exam is:

A's 77 to 100
B's 67 to 76
C's 57 to 66
D's 47 to 56

1) ANS $= \sum_{j=1}^{20} j^2 + \sum_{j=1}^{20} 2j = \frac{20\cdot 21\cdot 41}{6} + 20 \cdot 21 = 3290$. You did not have to simplify to 3290. Your grade for problem 1 is in the left margin. You may also see a note nearby, such as "$\sum = 14$", which you can ignore; it is just to help me add the grades.

2) Best is $\sum_{k=1}^{20} k^2 - \sum_{k=1}^{11} k^2 = \frac{20\cdot 21\cdot 41}{6} - \frac{11\cdot 12\cdot 23}{6} = 2364$. Simplification was not required. Some people tried this without using any summation formula but did not get 2364. There were inevitable calculation mistakes. A few people started with $\sum_{j=1}^{9}(j + 11)^2$ which is OK, but probably a little harder.

3) $\frac{1}{\ln(2)}$. I'd start with $2^x = e^{[\ln 2]x}$ and then set $u = [\ln 2]x$. There are other approaches.

4) $\pi/2$. Start with $\sin^2(x) = [1 - \cos(2x)]/2$ and then set $u = 2x$. As has happened so often in class, the $\cos(2x)$ term does not affect the final answer.

5) Use the abc-thm to remove the absolute values, $\int_{-1}^{0}(2 - x)^{1/2}\ dx + \int_0^2 (2 + x)^{1/2}\ dx$. Then two u-sub's, $-\int_3^2 u^{1/2}\ du + \int_2^4 u^{1/2}\ du = \frac{2}{3}(-u^{3/2}|_3^2 + u^{3/2}|_2^4) =$ etc.

6) xe^x using FTC.2 and the Chain Rule.

7) $\tan^{-1} x\ |_0^1 = \pi/4$.

8) By the definition of average value of $v(t)$, $\frac{\int_0^4 e^{-t}\ dt}{4} = \frac{1-e^{-4}}{4}$. Note that $\int e^{-t} = -e^{-t}$.

9) 7/3, but very few people got all the way to this with no errors. I gave decent partial credit for steps like $\Delta x = (2 - 1)/n = 1/n$ and $x_k^* = x_k = 1 + k/n$ and R.Sum $= \sum_{k=1}^{n}(1 + k/n)^2(1/n)$, etc.

After multiplying out, and applying summation formulas, the final step should look something like $\lim_{n\to\infty} 1 + (n + 1)/n + (2n^3 + \cdots)/6n^3 = 1 + 1 + 1/3 = 7/3$.

Several people were confused from the start about n. In some HW problems, n was given and you were asked to compute a Riemann sum to *approximate* an area. No limit required. See my noon class Exam I Key for such a problem. We will do more calculations like this late in Ch.7. In other problems, like this one, you need an exact answer and cannot choose a specific number for n. We will probably not do more like this - these are done mainly for understanding integration.

10) 20 sec. First, $v(t) - v(0) = \int_0^t a\ dt = 4t$, and $v(0) = 0$, so $v(t) = 4t$. Similarly, $s(t) = 2t^2$. Set $s(t) = 800$ and get $t = \sqrt{400} = 20$. You should simplify to 20 here because

it is so easy to do (1 point), but an answer like $\sqrt{403}$ could be left as it is.

11) FTFTT For the third one, consider $f(x) = |x+2|$, which is continuous, but not differentiable. It is easier to be integrable than differentiable. It may be harder to *calculate* integrals than derivatives.

12) See the text. Most people chose $\ln(ab)$. Common mistakes were not stating the theorem, not explaining all the steps, and not using equal signs. Several people started with $\int_1^{ab} \frac{dt}{t}$ without any comment on why (that it is the definition of $\ln(ab)$).

Bonus) Yes [one point for this]. The smallest and largest possible R.Sums are $\pm 2\pi \approx \pm 6$. Our heroes' answers are within those bounds, so they are possible. Some reasoning has been left to the interested reader, or ask me. No points for explanations that did not use $5 < 2\pi$ (other ideas are conceivable, but probably harder).

Remark: You might also be interested in the Exam 1 Key of my noon class. A couple of problems were similar, and I may have explained them a little differently on the two Keys.