MAC 2312 Sept 16, 2020
Exam I Prof. S. Hudson

Problems 1-7 are short answer, 7pts each. Show your work or comment on your reasoning. If any do not exist, explain. Simplify when that is reasonable (maybe not part 2).

1) $\sum_{k=1}^{10}\ 3k$

2) $\sum_{k=5}^{15} k^2$

3) $\frac{d}{dx}\int_2^x \sqrt{1+t^7}\ dt$

4) $\int_0^{\pi/2} \cos^2(x)\ dx$

5) $\int_0^5 \sqrt{25-x^2}\ dx$

6) $\int_{-1}^1 \frac{dx}{1+x^2}$

7) $\int_{-1}^1 \frac{1}{x^2}\ dx$

8) [10pts] Suppose the region between $y=\sqrt{\cos x}$ and $y=0$, with $0 \le x \le \pi/2$, is revolved around the x-axis. Use the disk method to find the volume.

9) [10pts] Compute the exact area under $y = 6x^2$ from $x = 2$ to $x = 3$ using Riemann sums. If you continue on the back, leave a note (as usual). Follow these rules:

9a) Use the right endpoint rule with regular partitions. Find the area of the k-th rectangle. You should get $(4+\frac{4k}{n}+(\frac{k}{n})^2)\frac{6}{n}$, but show all the work behind this formula.

9b) Write out the Riemann sum. Use summation formula(s) and a limit to find the exact area. Do not use an integral (except maybe to check your answer).

10) [7 pts] A rocket blasts off at time $t = 0$ (straight up, no initial velocity) with a constant acceleration of $a = 0.6$ miles /sec^2. How long will it take for the rocket to go 30 miles ?

11) [16 pts] Sketch the graph of $y = 2^x$. Then $y = \sec x$ separately. Use these to answer True or False to each part. You do not have to explain.

$3 < \int_0^2 2^x\ dx < 6$

$0 < \int_1^2 2^x\ dx < 3$

$3/4 < \int_0^{\pi/4} \sec x\ dx < 4$

$3/2 < \int_0^{\pi/2} \sec x\ dx < 40$

12) [8 pts] Choose ONE proof. The grading will be based on the clarity of your logic and explanations, as much as on any calculations involved. If you use the back of the page, leave a note here.

A. State and prove the FTC #1 (about $\frac{d}{dx}\int^x f$). This is mainly a calculation, but justify the steps.

B. State and explain in detail the general formula for volume in Ch.6.1. Roughly, it says $V = \int A(x)$. For maximum credit don't refer to disks - use slabs instead. Include a picture, a Riemann sum and several sentences.

Bonus: (5 points, maybe hard): The function $f(x) = |\cos x|$ is continuous on $[0, \pi]$ so it has an antiderivative $F(x)$ there such that $F(0) = 0$. Find a formula for $F(x)$. It should be in closed form (no integral signs, etc, in your answer).

Remarks, Answers: The average among the top 20 students was 63. The high scores were 89 and 85. The worst results were on problem 8 (24%) and the best were on problem 7 (91%). Here is an unofficial scale for the Exam:

A's 71 - 100
B's 61 - 70
C's 51 - 60
D's 41 - 50

1) $\frac{n(n+1)}{2}\,|_{n=10} = 165$. I did not give much credit for $3 + 6 + \cdots + 30$ unless you managed to add correctly. A summation formula is faster and more practical for long sums.

2) Since the sum starts at 5, not 1, it is "missing 4 terms" that add up to 30. Using some slightly non-standard FTC notation, $\frac{n(n+1)(2n+1)}{6}\,|_{n=4}^{n=15} = 1240 - 30 = 1210$. With this method, I did not require full simplification, but if you tried to add $5^2 + \cdots$ (which is not a good idea) you needed to get 1210.

3) $\sqrt{1+x^7}$ by the FTC #1.

4) $\pi/4$, using the usual identity, $\cos^2 x = \frac{1+\cos 2x}{2}$.

5) Based on a picture (a quarter circle), $A = \pi r^2/4 = 25\pi/4$.

6) $\pi/2$. The anti-derivative is $\tan^{-1} x$.

7) Does not exist. This function is not integrable because it is not defined at 0 and has an asymptote there. If you did not notice this, you probably got an answer of -2, but that cannot be right. Since $\frac{1}{x^2} > 0$, any answer would have to be positive.

8) $V = \int_0^{\pi/2} \pi \cos(x)\, dx = \pi$.

9a) Just the highlights: $\Delta x = 1/n$, $x_k = 2 + k/n$, height $= 6(2 + k/n)^2$, area of rect $= (4 + \frac{4k}{n} + (\frac{k}{n})^2)\frac{6}{n}$.

9b) Using summation formulas, lim R sum $= \lim(4n + \frac{4n(n+1)}{2n} + \frac{n(n+1)(2n+1)}{6n^2})\frac{6}{n} = 38$.

Some checked that $\int_2^3 6x^2\ dx = 2x^3|_2^3 = 54 - 16 = 38$, a good habit, but I did not grade it.

10) First IVP: from $a(t) = 0.6$ and $v(0) = 0$, get $v(t) = 0.6t$. Second IVP: from $v(t) = 0.6t$ and $s(0) = 0$, get $s(t) = 0.3t^2$. Set $0.3t^2 = 30$ and get $t^2 = 100$, so $t = 10$ seconds.

11) TTTF. There are various methods, but graphs seem the easiest, and the results here were mostly good. Use rectangles to bound the first three areas, above and below. But for $\int_1^2 2^x\ dx < 3$ use a trapezoid. The 4th one is not integrable.

12) See the textbook or lecture notes.

Bo) Note that $\cos x$ changes sign at $x = \pi/2$, so the formula for $F(x)$ will change there. This problem really has 2 parts.

$F(x) = \sin(x)$ for $0 \leq x \leq \pi/2$ and $F(x) = 2 - \sin(x)$ for $\pi/2 \leq x \leq \pi$.