Exam II
 MAC 2312

Oct 14, 2014
 S Hudson

Name

Show all your work. Use the space provided, or leave a note. Don't use a calculator or your own extra paper.

1-3 (10 points each) Set up each of these problems, by writing down the definite integrals that solve them. Show some work, such as a sketch, background formulas, and/or a brief discussion. But you do not have to evaluate them.

1) Find the arc length of the curve $y = 2x^3$, with $1 \le x \le 3$. Set-up only.

2) Find the volume of the solid that results when the region enclosed by $y = \sqrt{x}$ and $y = x$ is revolved about the line $x = 0$. Include a rough graph of the region and use shells. Set-up only.

3) A cylindrical water tank has radius 5 ft and is 9 ft tall. It is half-filled with water. How much work is required to pump all the water over the upper rim? Water weighs 62 lbs per cubic foot. Draw a picture and label the main variables. Set-up only.

4) [7pts] Express this integral in terms of u but do not evaluate it. $\int_0^1 e^{2x-1}\, dx$, $u = 2x-1$.

5) [10pts] Express the function $f(x) = \pi^{-x}$ as a power of e (find $g(x)$ so that $f = e^g$). Then compute $f'(x)$ and simplify if possible. Of course, it is OK to leave expressions such as $\sqrt{\pi}$ in your answer, not simplifying those.

6) [10pts] Find the volume of the solid generated when the region enclosed by the curves $y = \sqrt{x}$, $y = 2$ and $x = 0$ is revolved around the $y-$axis. Work this out completely, as usual.

7) [5pts] Compute $\int x \ln(x)\, dx$

8) [5pts] $\int \sin^4 x \cos^3 x\, dx$

9) [8pts] $\int \frac{dx}{x^2\sqrt{4-x^2}}$

10) [15pt] True or False:

For all $x > 0$, $\int_0^{e^x} \frac{dt}{t} = x$.

If each cross section of a solid S is a disk, then S is a solid of revolution.

Based on Hooke's Law, to stretch a spring 2 ft from its natural position requires 4 times as much work as stretching it only 1 ft.

The formula $\sin(a)\cos(b) = [\sin(a+b) + \sin(a-b)]/2$ is true for all a, b.

The partial fraction decomposition of $\frac{2x+3}{x^3}$ is $\frac{2}{x^2} + \frac{3}{x^3}$.

11) [10pts] Choose ONE, and explain thoroughly. You can answer on the back.

a) Explain the integral formula used in the Shell Method. Include: a picture, a limit, a sum, the volume of a shell, and explanation.

b) State, then prove, the Integration By Parts formula.

c) Derive (Prove) the reduction formula for $\int \tan^n x \, dx$.

BONUS: (5 pts) Use a picture to compute $\int_0^1 \sin^{-1}(x) \, dx$.

Remarks and Answers: The average was 61 out of 100 among the top 21, about the same as on Exam I. The high scores were 76 and 74, with almost all other scores in the [40,70] range. The lowest scores were on problems 5 and 9. For now, the scales are the same for Exam I and Exam II (and for the semester average):

A's = 73 - 100
B's = 63 - 72
C's = 53 - 62
D's = 43 - 52

1) $L = \int_1^3 \sqrt{1 + 36x^4} \, dx$, based on $f' = 6x^2$, etc.

2) $V = \int_0^1 2\pi x(\sqrt{x} - x) \, dx$. The limits of integration come from setting $\sqrt{x} = x$. A picture helps us choose a dx integral, and get height $= \sqrt{x} - x$ (if confused here, compare this with problems about the area between two curves). Several people did not read carefully, and used washers.

3) $W = (25\pi)62 \int_0^{4.5} 9 - y \, dy$. This is based on a y-axis that goes upward, with $y = 0$ at the bottom of the tank, which was the most common approach. Quite a few people confused *cylindrical* with *conical* which would be a bit harder. In that case, the answer might be $W = 62\pi \int_0^{4.5} (5y/9)^2 (9 - y) \, dy$, assuming the vertex of the cone is at the bottom, and that half-full means "4.5 feet deep" (which would not actually be my interpretation).

4) $\frac{1}{2} \int_{-1}^1 e^u \, du$

5) $f(x) = e^{-x \ln(\pi)}$, so by the Chain Rule, $f'(x) = -\ln(\pi) e^{-x \ln(\pi)} = -\ln(\pi)\pi^{-x}$.

6) With disks use dy, getting $V = \int_0^2 \pi(y^2)^2 \, dy = 32\pi/5$. Also, it's OK to use shells with $\int_0^4 2\pi x(2 - \sqrt{x}) \, dx =$ the same, though this method is a bit longer. I strongly suggest drawing a picture at the start, and thinking carefully about whether to use dx or dy before starting any calculations.

7) $x^2 \ln(x)/2 - x^2/4 + C$, from IBP. [typo corrected 10/13/16].

8) $\frac{\sin^5(x)}{5} - \frac{\sin^7(x)}{7} + C$

9) $\frac{-\sqrt{4-x^2}}{4x} + C$. The term $\sqrt{4-x^2}$ in the problem strongly suggests the trig substitution $x = 2\sin\theta$. This should be a knee-jerk reaction asap. The rest is moderately easy, ending with $\frac{1}{4}\int \csc^2(\theta)\ d\theta = \frac{-1}{4}\cot(\theta) + C =$ ans (from a triangle).

10) FFTTT

11) See the text or lectures. A slight majority chose b), IBP, though I imposed higher standards for clarity on that one. At a minimum, you should state the Product Rule (without any integrals yet) probably using the letters f and g. Then, explain that you are taking antiderivatives on both sides, etc. It is probably better NOT to clutter up your proof by discussing LIATE, specific examples, or the alternative u, v notation. Ideally, you should mention that f and g are assumed to have continuous derivatives, but omitting this point was forgivable.

B) Draw the rectangle where $0 \le x \le 1$ and $0 \le y \le \pi/2$, which has area $\pi/2$. The part of this that we do NOT want to include, in the upper left, is half of a sin bump, with area 1. So, ANS $= \pi/2 - 1$.

IBP should work too, but not with these instructions.