MAC 2312 | Oct 10, 2016
Exam II Key | Prof. S. Hudson

1) [5 pts] Find the volume of the solid generated when the region between $x = 4y - y^2$ and $x = 0$ is revolved around the y-axis. You can express your answer as a definite integral. You do not have to compute it.

2) [10 pts] Sketch the curve where $y = \sin^{-1}(x)$ for $0 \le x \le 1$ and find the area under it. Remark: this is also the region where $\sin(y) \le x \le 1$ and $0 \le y \le \pi/2$.

3) [10 pts] Find the volume of the solid generated when the region between the graphs of $f(x) = 1 + x$ and $g(x) = 2 + x$ over the interval $[0, 2]$ is revolved around the x-axis. State which method you are using. A graph or two is suggested.

4) [10 pts] Find the arc length of the curve $y = 3x^{3/2} - 1$ over the interval $[0, 1]$.

5) [10 pts] A conical water tank is 10 ft tall with a radius of 5 ft at the top. It contains water to a depth of 6 ft with density 62 lbs per cubic ft. How much work is required to pump the water over the top of the tank ? You can express your answer as a definite integral. You do not have to compute it.

6) [10 pts] Choose ONE. Explain the usual formula for one of these, with small extra credit for the second one. Draw a picture and label any relevant functions and/or intervals. Assume revolution around the x-axis. For full credit include a limit, a discussion of the 'pieces', a Riemann sum and, of course, *words.*

a) Volume with the Shell method.

b) Area of a surface of revolution.

7) [6 pts] Compute $\int e^x \cos(x)\ dx$.

8) [6 pts] Compute $\int \sin^3(x) \cos^4(x)\ dx$

9) [6 pts] Compute $\int_0^{\pi/4} \sqrt{9 - x^2}\ dx$

10) [6 pts] Compute $\int \ln(x + 1)\ dx$

11) [6 pts] Compute $\int \frac{dx}{x^2+4x+8}$

12) [15 pts] Answer True or False to each part. You do not have to explain.

If a cylinder S has height 2 and each horizontal cross section has area 5, then the volume of S is 10.

If a region R in quadrant 1 with area greater than 1 revolves around the x-axis, the resulting solid of revolution has volume greater than π.

One of the trig identities used in Ch.7 was $\sin(a)\sin(b) = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$.

If a region revolves around the y-axis, the Shell Method uses a dx integral.

The amount of work required to send a rocket infinitely (indefinitely) far from earth is infinite.

Bonus: (5 points, maybe hard): Fluid enters a hemispherical bowl with a radius of 10ft at a rate of 1/2 cubic ft per minute. How fast is the fluid rising when the depth is 5 ft?

Remarks and Answers: The average was 61, with highs of 90 and 84, which is fairly normal but a bit low. The scores averaged over 80% on problems 1, 5, 7 and 8 but under 50% on problems 2, 6, 10 and 11. Here is a scale for the exam:

A's 70 to 100
B's 60 to 69
C's 50 to 59
D's 40 to 49

To estimate your semester grade from your two exams scores, average them and put that on the scale below. I will include HW at the end, but it is hard to factor that in yet. See the syllabus about ± grades.

A's 69 to 100
B's 59 to 67
C's 49 to 57
D's 39 to 47

There were two versions of the exam, which differed slightly only on problems 8 and 11. The answers below match the questions above, but include remarks on the other version.

1) $V = \int_0^4 \pi(4y - y^2)^2 \, dy$ and stop.

2) The simplest solution (based on the remark and/or the picture) is to treat this as area between two curves, and get $\int_0^{\pi/2} 1 - \sin(y) \, dy = \pi/2 - 1$. The direct approach, $A = \int_0^1 \sin^{-1}(x) \, dx$, is harder to calculate, but you can go on with IBP.

3) 10π

4) $L = \int_0^1 \sqrt{1 + (9x^{1/2}/2)^2} \, dx = \cdots = (85\sqrt{85} - 8)/243$

5) $W = 62\pi \int_0^6 (y/2)^2(10 - y) \, dy$.

6) "Explain" means "justify" (not quite "prove") as done in class. See the text or lecture notes. Most people chose (a), but many did not know the Shell Method. A good

explanation should include a picture of a single shell with a fairly detailed discussion of its volume.

7) Using "the trick" as explained in class, $e^x(\sin(x) + \cos(x))/2 + C$.

8) Use $u = \cos(x)$. For this version of the exam, get $\cos^7(x)/7 - \cos^5(x)/5 + C$. The other version starts with $\cos^6$ instead of $\cos^4$ and then the answer is $\cos^9(x)/9 - \cos^7(x)/7 + C$.

9) Use $x = 3\sin(\theta)$ and $dx = 3\cos(\theta)d\theta$ to get to $9\int \cos^2\theta \; d\theta = \frac{9}{2}\int(1 + \cos 2\theta) \; d\theta$, etc. I realized a bit late that this doesn't simplify very nicely with $b = \pi/4$, so I gave full credit for getting this far.

10) $(x + 1)\ln(x + 1) - x + C$. There are several ways to start, but I'd set $u = x + 1$ (this kind of subn is almost never a bad idea) and get $\int \ln(u) \; du = u\ln(u) - u + C$, either from memory, or from IBP.

You can skip the u step and use IBP from the start, but that may get tricky (you may need to integrate $x/(x + 1)$ which is a fairly easy improper Ch.7.5 problem). Or, if you cleverly use $\int 1 \; dx = x + 1$, instead of x, the IBP is not too hard. Remember that integration may require some trial and error, so if you hit a dead end, try another approach.

11) Complete the square and then use $x + 2 = 2\tan\theta$, getting $\int((x + 2)^2 + 4)^{-1} \; dx = \int \frac{2}{4} d\theta = \frac{\theta}{2} + C = \frac{\tan^{-1}(\frac{x+2}{2})}{2} + C$.

If you had the other version of the exam, use $x^2 + 6x + 18 = (x + 3)^2 + 9$ with a very similar answer (change the 2s to 3s).

12) TFFTF

B) $1/150\pi$. See the bonus for Exam 2, Summer 2016.