Exam II and Key — Feb 25, 2016
MAC 2312 — S Hudson

1-3 (12 points each) Set up each of these problems, by writing down the definite integrals that solve them. Show some work, such as a sketch, background formulas, and/or a brief discussion. But you do not have to evaluate them.

1) Find the arc length of $y^2 = x^3$, $1 \le x \le 2$. Set-up only.

2) Find the volume of the solid that results when the region enclosed by $y = \sqrt{x-1}$, $y = x$, $y = 0$ and $y = 1$ is revolved about the x-axis. State at the start whether you are using disks, washers or shells. Set-up only.

3) Find the work done to pump the water out the top of this conical tank. The tank is 10 ft tall, and has a radius of 5ft (at the top). The water in the tank has a depth of 8 ft (so the top 2 ft is air). Recall that water weighs 62 lbs per cubic foot. Set-up only.

4) [7 pts] Compute $\int 3x \sin(x)\ dx$

5) [12 pts] Find the area between $y^2 = x$ and $y = x - 2$.

6) [6 pts] Compute $\int_0^{\pi/4} \sec^3 x \tan x\ dx$

7) [6 pts] Compute $\int_0^{\pi} \sin^2 3x dx$

8) [18 pt] True or False: Some are abbreviated from the HW. You can ask for clarification.

If a solid with volume V lies between $x = 2$ and $x = 5$ with cross-sectional area $A(x)$ then the average value of A on $[2, 5]$ is $V/3$.

Our Ch.5 definition of $\ln(x)$ is based directly on e^x.

The Integration By Parts formula is based directly on the Product Rule.

The functions $f(x) = \ln(3x)$ and $g(x) = \ln(\pi x)$ differ by a constant.

The arc length of the curve $y = x^3$ from $x = 1$ to $x = 4$ is $L = \sqrt{8}$.

The reduction formula for $\int \sin^n x\ dx$ is $\cos^{n-1} x \sin x + \frac{1}{n} \int \cos^{n-2} x\ dx$

9) [15pts] Choose ONE proof. For a or b, include a picture, a limit, a sum, and words.

a) Derive the integral formula used to compute arc length.

b) Derive the Washer Method formula for volume.

c) Derive the reduction formula for $\int \sin^n x\ dx$.

BONUS: (5 pts) [typo corrected 2/28/16] Two important functions that we did not cover are the hyperbolic sine and cosine functions, $\sinh(x)$ and $\cosh(x)$. Compute and simplify

$\frac{d}{dx}\sinh(x)$ using only the two identities below. Leave your answer in terms of $\sinh(x)$ and/or $\cosh(x)$. No credit for memorized answers or guesses.

$$\sinh(x) + \cosh(x) = e^x \quad \text{and} \quad \cosh^2(x) - \sinh^2(x) = 1$$

Remarks and Answers: The average among the top 22 was 66 out of 100, which is pretty normal. The high score was 88, followed by three 84's. The results were similar on all problems except for a 47% average on problem 7 and a 92% average on problem 1. Here is an estimated scale for Exam 2:

A's 74 to 100
B's 64 to 73
C's 54 to 63
D's 44 to 53

As usual, you can apply $\pm$'s by dividing each range into thirds. For example 71 to 73 is a B+. The current scale for the semester is halfway between the one above and the one on the Exam 1 Key (so, for example, the A's start at 79). I do not have enough information yet to include your HW or other tweaks into the scale.

If you are undecided about dropping you can see me. As a rough guide, I think that if your average is below 50 now, you have poor chances to pass with a C. If you are in the 50 to 60 range, your odds are not very good, but you can probably pass with a strong effort.

Many of the problems on this exam can be solved several ways. In general, I gave full credit for any valid method, if you showed your work and got the right answer. But for most of the problems there is a standard approach, which is easier than the others, and I gave more partial credit for that method if your final answer was wrong. For simplicity, this key focuses only on the standard methods.

1) $L = \int_1^2 \sqrt{1 + \frac{9x}{4}}\, dx$. I did not require a picture for this one because it is so easy without one.

2) Use shells, $V = \int_0^1 2\pi y(y^2 + 1 - y)\, dy$.

In general, people who drew a fairly accurate picture did the best. The region is roughly a parallelogram, except that the line from (1,0) to (2,1) is curved. The picture shows that shells with dy work best and that y varies from 0 to 1. Or, if you really want to use dx, it shows that x varies from 0 to 2, with a change of pattern at $x = 1$.

3) $W = 62\pi \int_0^8 (y/2)^2(10 - y)\, dy$ and perhaps simplify it.

4) $-3x\cos x + 3\sin x + C$ [typo corrected 6/12/16]

5) $A = \int_{-1}^2 y + 2 - y^2\, dy = 9/2$. Again, draw a careful picture (see remarks on problem 2), to choose dy and to see that $y + 2 > y^2$. Find the limits by solving $y + 2 = y^2$.

6) $(2\sqrt{2}-1)/3$. This should be easy but it requires several basic skills such as $u = \sec(x)$ (see Ch.5.9 and Ch.7.3) and $\sec(\pi/4) = \sqrt{2}$. If you have not memorized that, you should recall that $\cos(\pi/4) = 1/\sqrt{2}$ and that $\sec(x) = 1/\cos(x)$.

7) $\pi/2$, using $\sin^2(3x) = (1-\cos(6x))/2$. You can use $u = 3x$ instead at the start, or use $u = 6x$ later. Or, just state that $\int_0^\pi \cos(6x)\ dx = 0$, if you understand and trust that shortcut.

8) TFTTFF

9) See the text or lectures. Most people chose b, and it seemed that those who studied it did OK, if not great. Ideally, you should include a picture of a single washer and discuss its volume, including Δx. Also, $V = \pi \int_a^b R^2 - r^2\ dx$ is pretty informal (though I included it in a lecture); $V = \pi \int_a^b f(x)^2 - g(x)^2\ dx$ is better.

B) Nobody got this. You need to play around with the given info. For example, see if you can justify these steps:

$\cosh(x) - \sinh(x) = [\cosh^2(x) - \sinh^2(x)]/[\cosh(x) + \sinh(x)] = e^{-x}$
$\sinh(x) = \frac{e^x - e^{-x}}{2}$
$\cosh(x) = \frac{e^x + e^{-x}}{2}$
$\frac{d}{dx}\sinh(x) = \cosh(x)$

The original problem had a typo (in effect, sinh and cosh were interchanged). In that case, a similar calculation leads to the same conclusion. If interested, these functions are covered near the end of Ch.6, the section about hanging cables.