MAC 2312 June 8, 2016
Exam II Prof. S. Hudson

1) [10 pts] Sketch the region enclosed by $x = \sin(y)$, $x = 0$, $y = \pi/4$ and $y = 3\pi/4$, and find its area.

2) [10 pts] Find the volume of the solid generated when the region between the graphs of $f(x) = 1/2 + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved around the x-axis.

3) [7 pts] Find the volume of the solid generated when the region between $y = 2x - x^2$ and $y = 0$ is revolved around the y-axis. Set-up only - you do not have to compute the integral. State which method you are using. A graph is suggested.

4) [10 pts] Find the exact arc length of the curve $y = 3x^{3/2} - 1$ over the interval $[0, 1]$.

5) [10 pts] Find the work required to lift an astronaut to a point 800 miles above the surface of the earth.

You can assume he weighs 150 lbs at sea level (4000 miles from earth's center), and in general weighs k/x^2 lbs when he is x miles from earth's center (and $x - 4000$ miles above earth). You should try to compute k, but if you cannot, leave it in your answer for partial credit.

6) [5 pts] Compute $\int x^2 \cos(x)\ dx$.

7) [5 pts] Compute $\int \sin(3x) \cos(2x)\ dx$

8) [5 pts] Compute $\int \sin(\ln x)\ dx$

9) [5 pts] Compute $\int \tan^2 x \sec^4 x\ dx$

10) [5 pts] Compute $\int \frac{\sqrt{x^2-9}}{x}\ dx$

11) [5 pts] Start the partial fraction method for $\int \frac{3x-1}{(x-3)(x+4)}\ dx$. You can stop after writing the form of the decomposition. You do not have to compute A, B, etc or find any antiderivatives.

12) [5 pts] Estimate $\int_4^8 f(x)\,dx$ using T_4 (the trapezoid rule with $n = 4$) and $y = f(x) = x^{-1/2}$. If you cannot do that, you can estimate it using M_2 instead, for partial credit. You can use any data from this table that you need:

x	y
2	0.71
3	0.58
4	0.50
5	0.45
6	0.41
7	0.38
8	0.35
9	0.33

13) [10 pts] Answer True or False to each part. You do not have to explain.

If each cross section of a solid S is a disk, then S is a solid of revolution.

If the displacement of a particle is negative then its average velocity cannot be positive.

If a region revolves around the y-axis, the Washer Method uses a dx integral.

Applying the LIATE method to evaluate $\int x^3 \ln(x)\,dx$, we should set $u = x^3$ and $dv = \ln x\,dx$.

It follows from Hooke's law that to triple the distance a spring is stretched, 9 times as much work is required.

14) [8pts+] Choose ONE proof. If possible, use sentences or formulas with complete justifications. These are listed in estimated order of difficulty. You may get more points for choosing B than A, etc.

A. [max 8 points] State and prove the integration by parts formula.

B. [max 10 points] State and prove the Shell method formula. Include at least 2 pictures, 2 formulas from geometry, a Riemann sum, and a limit.

C. [max 12 points] Derive the reduction formula for $\int \tan^n x\,dx$.

Bonus: (5 points, maybe hard): Fluid enters a hemispherical bowl with a radius of 10ft at a rate of 1/2 cubic ft per minute. How fast is the fluid rising when the depth is 5 ft?

Remarks and Answers: The average among the top 22 was 64, with high scores of 81 and 80. The average results on the 10 point problems were generally above 60%, but on the 5 points ones (except for # 11) it was more like 50%, maybe less.

A's 72 to 100
B's 62 to 71
C's 52 to 61
D's 42 to 51

To estimate your semester grade, average your two exam scores and use the scale below. If your HW average is much different from your exam scores (differing by 20 points, for example) you might adjust your average about ± 2 points, or see me for help with that. Your semester grade will eventually also include Exam 3 and the Final. Those might change your grade as much as one letter (from a D+ to a C+, for example) but rarely more than that.

A's 74 to 100
B's 64 to 73
C's 54 to 63
D's 44 to 53

1) $\sqrt{2}$ from $\int_{\pi/4}^{3\pi/4} \sin(y)\ dy$.

2) $69\pi/10$. As usual, a good picture made a good start. It was not required, but it influenced partial credit. Mainly you need to draw the parabola above the straight line and recognize that washers are needed (using shells might be possible but much much harder). Then $V = \pi \int_0^2 (1/2 + x^2)^2 - (x)^2\ dx = \cdots = 69\pi/10$.

3) Use shells and dx. $V = \int_0^2 2\pi x(2x - x^2)\ dx$. Stop.

4) $L = \int_0^1 \sqrt{1 + 81x/4}\ dx = \cdots = [85^{3/2} - 8]/243$.

5) $W = \int_{4000}^{4800} \frac{k}{x^2}\ dx = 2.4 \times 10^9 (\frac{1}{4000} - \frac{1}{4800})$. The formula $x - 4000$ was intended to clarify the problem, but is not part of the solution. There were mistakes about x - when to replace it by 4000 (when finding k) and when not to (when integrating Force).

Remarks on grading, 6 to 12: I usually give less partial credit on short 5 point problems, especially with multiple mistakes. I did not give much partial credit for starting off in the wrong direction (eg a legal step that does not actually help towards the answer). In general, forgetting a $+C$ cost one point, but I tried not to deduct more than 3 points total for this, for the whole exam.

6) $x^2 \sin(x) + 2x \cos(x) - 2\sin(x) + C$, from IBP twice. Most people started well, but there were many calculation errors. Using a table might have helped with that.

7) $-[\cos x + \cos 5x]/2 + C$, from the trig identity for $\sin(a)\cos(b)$.

8) $[x\sin(\ln(x)) - x\cos(\ln(x))]/2 + C$, from IBP twice and adding ANS to both sides.

9) $\tan^3(x)/3 + \tan^5(x)/5 + C$, from $u = \tan(x)$.

10) $\sqrt{x^2 - 9} - 3\sec^{-1}(x/3) + C$. Start with $x = 3\sec(\theta)$ and draw a triangle at the end to simplify the answer. Some people started with a triangle, and soon got to $x = 3\sec(\theta)$, but their overall success rate seemed a bit less than average.

11) $\frac{A}{x-3} + \frac{B}{x+4}$. It is also OK to reverse the A, B or to include the integral sign, etc. Yes, this was very easy, but it was about a very recent section.

12) Approx 1.66. For T_4 use the table with x_k going from 4 to 8, and $\frac{b-a}{2n} = \frac{1}{2}$. For M_2 use the table with x=5 and x=7 only, and $\frac{b-a}{n} = \frac{4}{2} = 2$.

13) FTFFT

14) See the text or lecture notes. Most people chose A and did pretty well. The most common problem was weak explanation. I considered the $\int u\, dv$ version optional, and did not grade that, if the $\int f'g\, dx$ version was OK.

Almost as many chose B, also with mostly good results. The best answers had a clearly labeled picture of a shell and did not use '$A(x)$' which makes more sense in the Disk/Washer Methods. For most pictures drawn, the formula should include $\int_0^b$. But $\int_a^b$ is also OK, with different pictures. Likewise, you can answer this using either dx or dy, but it should match your picture / introduction.

The results were less good on C. The best way to start (which was not very obvious) was $\int \tan^n x\, dx = \int \tan^{n-2} x[\sec^2 x - 1]\, dx$, etc.

B) Nobody got this one. It is basically a related rates problems (Calc I), but with volume included. Here is an outline (not checked carefully). Let $y(t)$ be the depth of the water, $0 \le y \le 5$. Then $dy/dt = (dV/dt)/(dV/dy) = (1/2)/75\pi = 1/150\pi$. This is based on $V = \int_0^y \pi(10^2 - (10-z)^2)\, dz$ (from the disk method and Pythagoras) followed by FTC#2 and setting $y = 5$. This is ex.6.2.56. See also 6.2.55, though my solution does not use 55 directly.

Most of the problems on the exam are from textbook exercises and Examples such as 6.1.11, 6.2Ex4, 6.3.10, 6.4.3, 6.6Ex4, 7.2.7, 7.2.21, 7.2.64a, 7.3 Ex4, 7.3.13, 7.4.7 and 7.5.1.