Exam II | March 2, 2017
MAC 2312 | S Hudson

The first problems are worth 6 points each.

1) Find the area of the region which is bounded by $x = y^2$ and $y = x - 2$.

2) Compute $\int \frac{dx}{x^2+4x+5}$

3) Compute $\int x \ln(x)\ dx$

4) $\int \sin^6 x \cos^3 x\ dx$

5) Compute $\int \frac{x+1\ dx}{x^2-9}$.

6) Compute $\int e^x \sin(x)\ dx$.

7) [9pts] Compute $\int \sin(3x) \sin(4x)\ dx$

8) [10pts] A chain, lying on the floor, is 30 ft long and weighs 20 lbs per foot (600 lbs total). Find the work required to lift it and hang one end from the ceiling, which is 40 feet above the floor.

9) [10pts] Find the arc length of the curve $y = x^{2/3}$, with $1 \le x \le 8$. Work this out to a number, as usual.

10) [10pts] Find the volume of the solid that results when the region enclosed by $y = \sqrt{x}$ and $2y = x$ is revolved about the line $x = 0$. Include a rough graph of the region and use shells. Set-up only.

11) [15pt] True or False:

The work required to pump water into an empty conical tank from the bottom equals the work required to pump the water out of a full tank over the top.

If a region revolves around the y-axis, the Washer Method uses a dx integral.

Based on Hooke's Law, to stretch a spring 3 ft from its natural position requires 3 times as much force as stretching it only 1 ft.

The formula $\sin(a)\cos(b) = [\sin(a+b) + \sin(a-b)]/2$ is true for all a, b.

If $\frac{3x^2-x+5}{x^3+x^2+2x+2} = \frac{A}{x+1} + \frac{Bx+C}{x^2+2}$ for all x, then $A = 2$.

12) [10pts] Choose ONE, and explain thoroughly.

a) Explain the integral formula used in the Shell Method. Include: a picture, a limit, a sum, the volume of a shell, and explanation.

b) Explain (as in (a) above) the integral formula for arc length.

c) Derive (Prove) the reduction formula for $\int \sin^n x \, dx$.

BONUS: (5 pts) Compute $\int_0^1 \cos^{-1}(x) \, dx$.

Remarks and Answers: The average among the top half was 65, with highs of 97 and 78. The grades were slightly low on problems 1 and 5, slightly high on 3 and 6. The scale for the exam is the same as Exam I. You can estimate your semester grade so far by averaging your two exam scores and using the same scale. See me if you want help including your HW into this estimate.

A's 74 to 100
B's 64 to 73
C's 54 to 63
D's 44 to 53

1) $A = \int_{-1}^{2} (y+2) - y^2 \, dy = \cdots = 9/2$. The most common mistakes were in sketching the region, especially below the x-axis. See Ch.6.1 Ex 4 (which uses dx and takes longer) or Ex 5 (which uses dy, as I am doing here).

2) Complete the square, then a substitution. So, $\int \frac{dx}{(x+2)^2+1} = \cdots = \tan^{-1}(x+2) + C$.

3) $\frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$. Use IBP with $g(x) = \ln x$.

4) $\frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C$. Use $u = \sin x$.

5) $\frac{1}{3} \ln|x+3| + \frac{2}{3} \ln|x-3| + C$. Use partial fractions. A few people tried $x = 3\sec\theta$, but this is harder and they didn't finish.

6) $\frac{e^x(\sin x - \cos x)}{2} + C$. Use IBP twice and "the trick".

7) $\frac{\sin(x)}{2} - \frac{\sin(7x)}{14} + C$.

8) 15,000 ft-lbs. The most common problem was the planning - inconsistencies between the picture and the integral, between the domain of the variable and the formula for distance. There are several approaches to this problem. You should settle on one you like, draw a picture or two, and clearly define a variable or two. Making this clear to me also helps with partial credit.

a) If you draw the "before picture" of the chain stretched out on the ground, you will probably use $0 \le x \le 30$ to refer to locations in the chain at the start. If you pull on the left end of the chain (where $x = 0$), you should get to something like $W = \int_0^{30} 20(40-x) \, dx$, etc. If you pull on the other end (where $x = 30$) you would replace the $40 - x$ by $x + 10$, but should get the same answer either way.

b) If you draw the "after picture" of a hanging chain, and set $y = 0$ at ground level, then distance is simply y, and you should get to something like $W = \int_{10}^{40} 20y\ dy$.

Notice that you need to make a few decisions at the start, about how to tell the story, before writing out W. Get a clear picture of that in your head and on paper. Do not change your mind halfway through. Do not rely too much on memory of similar problems (which might not exactly fit your chosen plan). Of course, you can change plans if your first one fails, but do not mix two plans. For practice, you can check that the 3 formulas for W in this answer key are equivalent to each other through simple u substitutions. But this insight is not important for solving the problem. You just need one solid plan.

9) $L = \int_1^8 \sqrt{1 + (\frac{2x^{-1/3}}{3})^2} dx = \int_1^8 \sqrt{1 + \frac{4x^{-2/3}}{9}} dx$. Most people got this far, but no further, and got 6 points. The next step is hard to find, but we did this example in class, and I was hoping most people would remember it. The algebra is based on the fact that $x^{-1/3}\sqrt{x^{2/3}} = 1$.

$L = \int_1^8 x^{-1/3}\sqrt{x^{2/3} + \frac{4}{9}}\ dx$. Now, set $u = x^{2/3} + \frac{4}{9}$ and the rest is routine.

This is Ex.1 of Ch.6.4, with x and y interchanged (and also ex.6.4.5). The solution above is the rather hard "solution b". But remember that many problems can be approached several ways. You can solve this using the simpler "solution a", though nobody tried that on this exam.

10) $V = \int_0^4 2\pi x(\sqrt{x} - x/2)\ dx$. Partial credit mainly for the limits of integration and that height $= \sqrt{x} - x/2$.

11) FFTTF

12) See the text or lectures. Most people chose A and did OK. For full credit I wanted a picture of a shell and a couple of sentences to guide the reader.

B) A picture shows that the area is $\int_0^{\pi/2} \cos(y)\ dy = 1$. This change of variable is similar to the idea of problem 9, solution "a". Nobody did this.

Alt.soln: Integration by parts eventually leads to $x \cos^{-1} x - \sqrt{1 - x^2}\ |_0^1 = 1$. Several people started on this longer path, and some got partial credit, but nobody quite finished. Try to expand your toolkit. Consider several strategies for an example before starting to calculate.