Exam II — Noon Feb 27, 2018
MAC 2312 — S Hudson

1-3 (8 points each) Set up each of these problems, by writing down the definite integrals that solve them. Show some work, such as a sketch, background formulas, and/or a brief discussion. But you do not have to evaluate them.

1) Find the arc length of $y^2 = x^3$, $1 \le x \le 2$. Set-up only.

2) Find the volume of the solid that results when the region enclosed by $y = \sin(x)$, $y = 0$, with $0 \le x \le \pi$ is revolved about the y-axis. State at the start whether you are using disks, washers or shells. Set-up only.

3) Find the work done to pump water *into* this conical tank, from the bottom, to a depth of 5 ft. The tank is 9 ft tall, and has a radius of 3ft (at the top). Recall that water weighs 62 lbs per cubic foot. Set-up only.

4) [10 pts] Let R be the region bounded by $y = 4 - x^2$, $y = 0$ and $x = 0$. Compute the volume of the solid generated by revolving R around the x-axis. Your final answer (also for problems 5 and 6) should be a simplified number, such as $3\pi/4$.

5) [10 pts] Find the area enclosed between $y = x^2$ and $y = 3x$.

6) [10 pts] Compute $\int_0^{\pi/4} \sec^4 x \tan^2 x \; dx$

7) [10 pts] Compute $\int \sin^{-1}(x)dx$

8) [18 pt] True or False: Some are abbreviated from the HW. You can ask for clarification.

The surface area of a sphere of radius r (in feet^2) is less than its volume (in feet^3).

The formula $\sin(a)\sin(b) = [\cos(a+b) - \cos(a-b)]/2$ is true for all a, b.

A driveway must do work against gravity to support a car on it.

The Integration By Parts formula is based directly on the Product Rule.

The functions $f(x) = \log_2(3\sin^2 x)$ and $g(x) = \log_2(\pi \sin^2 x)$ have the same derivative.

The work required to pump water into a conical tank from the bottom (and fill it) equals the the work required to pump the same water out the top.

9) [8 pts] $\int \frac{dx}{1+9x^2}$. Hint: use a trig substitution such as $x = \frac{1}{4}\sin\theta$ (but that one is not correct).

10) [10 pts] As you know, $e = \lim_{x\to\infty}(1 + 1/x)^x$. Use this and a substitution, to compute $\lim_{x\to\infty}(1 + 3/x)^{3x}$.

BONUS: (5 pts) Derive the integral formula used to compute arc length. Pay special attention to the picture and the steps involving Δx (the grading will be based mostly on that part, and not much partial credit is likely).

Remarks, Scale, Answers: The average among the top 24 was 58%, which is a little low, with high scores of 87 and 75. The average score on each problem was approx 50% to 70%, except for problems 1,2,3 and 5 (75%) and 10 (15%). Here is an advisory scale for Exam II:

A's 68 to 100
B's 58 to 67
C's 48 to 57
D's 38 to 47

You can estimate your letter grade for the semester by averaging your two letter grades on the exams. Or you can average your two numerical scores and put that on the scale below. I include the HW scores at the end.

A's 70 to 100
B's 60 to 69
C's 50 to 59
D's 40 to 49

1) $L = \int_1^2 \sqrt{1 + 9x/4}\; dx$. Note $y = x^{3/2}$ and $y' = 3x^{1/2}/2$. So, $[y']^2 = [3x^{1/2}/2]^2 = 9x/4$. You should do that last easy simplification.

2) Using shells, $V = \int_0^{\pi} 2\pi x \sin x\; dx$. In principle, you could use washers, but that is much harder. Nobody got it that way, and I did not give much partial credit for trying that method. You cannot use disks.

3) If you set $y = 0$ at the bottom and $y = 9$ at the top (which is the normal approach), then $W = 62\pi \int_0^5 (y/3)^2 y\; dy$. Here $r = y/3$ and distance $= y$. Other correct answers are possible in principle, but I don't recall seeing any. I did give full credit for using x instead of y, just a minor notational difference.

4) I intended for the region to be *in the first quadrant*. In that case, $V = \int_0^2 \pi(4-x^2)^2\; dx = 256\pi/15$ using the disk method. If you included the second quadrant, which is also quite reasonable, the answer is $V = \int_{-2}^2 \pi(4-x^2)^2\; dx = 512\pi/15$ (exactly twice as big). Both answers got full credit.

Some common errors were $\int_0^2 \pi(x^2-4)\; dx = \cdots = 16\pi/3$ and $\int_0^2 \pi(4-x^2)^2\; dx = \int_0^2 \pi(16-x^4)\; dx \cdots = 128\pi/5$. While it is possible to use Shells (with dy), it is much harder. In a practical sense, shells are wrong for this example.

5) $A = \int_0^3 3x - x^2\; dx = 9/2$. Or, $A = \int_0^9 (\sqrt{y} - y/3)\; dy = 9/2$.

6) Set $u = \tan x$ and get $8/15$. This is the recommended method and the simplest one. Using trig identities (only) did not turn out well.

7) $x \sin^{-1} x + \sqrt{1-x^2} + C$ from "solo IBP", with $f'(x) = 1$, etc.

8) FFFTTF see me if needed.

9) Use $x = \frac{1}{3}\tan\theta$. Get $\frac{1}{3}\tan^{-1}(3x) + C$. A somewhat common mistake was a "substitution" like $x = \frac{1}{3}\tan x$, but a true substitution must involve two different variables. This mistake is serious, not just notational, and it does not lead to the right answer.

10) e^9, using $3t = x$ (you can use u or any letter instead, but this is not the same "u-substitution" often used with integrals). The average score here was pretty low, as if this problem was 'out of the blue,' but see HW 5.10.11.

B) See the lecture notes or the text. Include a triangle with two sides labeled Δx and Δy (and some fairly routine algebra, some words, etc).