Exam II 2pm, Mar 1, 2018
MAC 2312 S Hudson

1-3 (8 points each) Set up each of these problems, by writing down the definite integrals that solve them. Show some work, such as a sketch, background formulas, and/or a brief discussion. But you do not have to evaluate them.

1) Find the arc length of $y = 4x^2$, $1 \leq x \leq 2$. Set-up only.

2) Find the volume of the solid that results when the region enclosed by $y = \sqrt{x}$, $y = x$ and $y = 0$ is revolved about the $y$-axis. State at the start whether you are using disks, washers or shells. Set-up only.

3) Suppose a rocket weighs 15 tons, including fuel, before launch. It burns one half ton of fuel per 100 miles of flight. How much work is done, as the rocket pushes itself 1000 miles into the sky ? You may ignore Newton's law, that the force of gravity decreases during the flight, and you can answer in units of mile-tons (so, unless you do something unexpected, you can just ignore units). Set-up only.

4) [10 pts] Let $R$ be the region bounded by $y = 2 - x$, $y = 0$ and $x = 0$. Compute the volume of the solid generated by revolving $R$ around the $x$-axis. Your final answer (also for problems 5 and 6) should be a simplified number, such as $7\pi/4$.

5) [10 pts] Find the area enclosed between $y = x^3$ and $y = 4x$.

6) [10 pts] Compute $\int_0^{\pi/4} \sec x \tan^3 x \, dx$

7) [18 pt] True or False:

For all real numbers $x$, $\int_1^{e^{2x}} \frac{dt}{t} = 2x$.

If $L = \int_1^8 \sqrt{1 + (f'(x))^2} \, dx$ is an arc length, then $L > 6$.

Based on Hooke's Law, to stretch a spring 2 ft from its natural position requires 4 times as much work as stretching it only 1 ft.

The formula $\sin(a)\cos(b) = [\sin(a+b) - \sin(a-b)]/2$ is true for all $a, b$.

$\int \cot x \, dx = \ln|\cos x| + C$

A good start on $\int_0^1 \sqrt{4 - x^2} \, dx$ would be $x = 2\sin\theta$.

8) [8 pts] Begin this partial fractions example, including the step with $A$, $B$ etc in the numerators, but stop there. Don't compute $A$, $B$ etc: $\int \frac{x}{(x^2+4)(x-1)^2} \, dx = ?$

9) [10 pts] Compute $\int e^x \sin x dx$ using IBP.

10) [10 pts] Choose ONE and explain the main formula thoroughly, probably including a picture or two and a Riemann sum.

A) The Washer method (to set this up, draw and label a typical region including an $f$ and a $g$, and revolve the region around the $x$ axis).

B) The arc length formula (you can assume $y = f(x)$ for $a \leq x \leq b$).

BONUS: (5 pts) List these 3 numbers in increasing order; $p = \ln 10$, $q = \sum_{k=1}^{9} 1/k$ and $r = \sum_{k=2}^{10} 1/k$. Justify your answer.

**Remarks, Scales and Answers:** The average was 77 based on the top 25 scores, which is fairly high. The best scores were 94 and 90. The grades were good on all problems except maybe problem 3 (55%). Here is an advisory scale for Exam II:

A's 85 to 100
B's 75 to 84
C's 65 to 74
D's 55 to 64

You can estimate your letter grade for the semester by averaging your two letter grades on the exams. Or you can average your two numerical scores and put that on the scale below. I include the HW scores at the end.

A's 81 to 100
B's 71 to 80
C's 61 to 70
D's 51 to 60

1) $L = \int_1^2 \sqrt{1 + 64x^2}\; dx$.

2) You can use Shells and dx, $V = \int_0^1 2\pi x(\sqrt{x} - x)\; dx$ (tall rectangles). Or, you can use Washers and dy, $V = \int_0^1 \pi(y^2 - y^4)\; dy$ (wide rectangles). You cannot use Disks on this solid.

The most common problem was confusion, over dx vs dy, over the larger function vs the smaller function, etc. Again, I suggest drawing a picture of the region, and a thin rectangle inside it as a guide. Most people who did that got it right.

Don't try to use Shells and dy on this one. Generally such answers did not get much credit. See the Note to problem 4.

3) $W = \int_0^{1000} 15 - \frac{y}{200}\; dy$. Other correct answers are possible, but I didn't see any.

This follows the common "chop-the-path" pattern used with most spring and rocket examples. So $W = \int F$, where $F$ is the weight. That decreases from 15 when $y = 0$ to 10 when $y = 1000$. With a little algebra, $F(y) = 15 - \frac{y}{200}$. Here, I am using $y$ for distance traveled, so $0 \leq y \leq 1000$.

A few people tried to use Newton's Law $F = c/y^2$, though you were advised not to. It makes the problem harder and it is not very important for small altitudes. If you really want to use it, then let $4000 \le y \le 5000$ instead, as in the lecture example. You get something like $W = \int_{4000}^{5000} c[15 - \frac{(y-4000)}{200}]/y^2\ dy$.

4) The standard method : use Disks, $V = \int_0^2 \pi(2-x)^2\ dx = \cdots = 8\pi/3$. Shells with dy are also possible, but Washers are not. See the suggestion in the answer to 2) about choosing your method.

Note: A common mistake was to revolve the region around the $y$-axis. In that case, the answer would be $V = \int_0^2 \pi(2-y)^2\ dy = \cdots = 8\pi/3$. The same! But this was mostly luck, and it got only partial credit. A few people wrote $V = \int_0^2 \pi(2-y)^2\ dy = \cdots = 8\pi/3$ without remarks or a picture, and I assumed they made this same mistake.

5) The question was unclear on whether to include the third quadrant, so I accepted either way. Most people did the 1st quadrant only, with $A = \int_0^2 4x - x^3\ dx = 4$. A few people included the third quadrant area too (perhaps that is more correct), and by symmetry they got $A = 8$. Both answers got full credit.

It is not OK to use $A = \int_{-2}^2 4x - x^3\ dx = 0$, which counts half the area as negative, a mistake. It is fairly common for an integral to equal 0, but not for such an area be zero. If you want to use $A = \int_{-2}^2 |4x - x^3|\ dx$ you may use the abc theorem on it, and should get 8, but using symmetry is easier for this example.

6) Std method: set $u = \sec x$ (because the 3 is odd) and get $(2-\sqrt{2})/3$. I don't know another standard method for this. One student found a nice trick (but didn't finish it):

$\int_0^{\pi/4} \sec x(\sec^2 x - 1)\tan x\ dx = \int_0^{\pi/4} \frac{\sin x}{\cos^4 x} - \frac{\sin x}{\cos^2 x}\ dx = \int_1^{\sqrt{1/2}} \frac{-1}{u^4} + \frac{1}{u^2}\ du$, etc, leading to the same answer.

7) TTTFFT See me about these, if needed.

8) $\frac{Ax+B}{x^2+4} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$

9) $A = e^x(\sin x - \cos x)/2 + C$. This is 'The Trick', of using IBP twice and then solving for $A$.

10) See the text or the lectures. The grading of a proof is somewhat subjective, but when grading (A) for example, I was looking for key symbols, words and ideas such as

$\sum$, lim, $\Delta x$, approximate, a picture of a washer, the volume of a washer, etc.

Bonus) $r < p < q$. Reasoning: $p = \int_1^{10} \frac{dt}{t}$ by definition of ln(10). And $q$ is a LER Riemann sum for that, $q = L_9$. Since $y = 1/t$ decreases, $q > p$. Likewise, $r = R_9 < p$.

We will use summations to approximate numbers like ln(10) in Ch.7.7 and Ch.9. But there is much more to the story, especially in Ch.9. Those summations are not quite like these.