MAC 2312 Oct 15, 2020
Exam II Prof. S. Hudson

Problems 1-6 are 7pts each. Other unlabeled problems are 10 points each. You can use the formulas below for any problems on this exam, as needed.

a) $\int \frac{dx}{1-\cos ax} = -\frac{1}{a}\cot(\frac{ax}{2}) + C$

b) $\int \tan^n x\ dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\ dx$

c) $\int \frac{x^2}{\sqrt{a^2-x^2}}\ dx = \frac{a^2}{2}\sin^{-1}(\frac{x}{a}) - \frac{x}{2}\sqrt{a^2-x^2} + C$

1) $\int_0^\pi \sin^2 x\ dx$

2) $\int \sec^3 x \tan x\ dx$

3) $\int \tan^4 x\ dx$

4) $\int \frac{x^2}{\sqrt{4-x^2}}\ dx$

5) $\int \ln x\ dx$

6) $\int \frac{x+3}{x+1}\ dx$

7) Find the arc length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$. Of course, you can express the relation as $2y^{3/2} = x$ if you like.

8) Use an integral as in Ch 6.4 to find the surface area of the sphere of radius 2 centered at the origin. [Do not use the standard formula for the area.]

In each of the next 2 problems, answer with an integral. You do not have to evaluate them.
9) This is a slight variation on Ch.6.5 Ex.5. A conical tank is 10 ft tall and has radius 5 ft at the top (so, in the notation from class, $r = y/2$). It is filled to a depth of 7 ft with water weighing 62 lbs per cubic ft. How much work is required to pump the water to the rim at the top ? Answer with an integral.

10) The region where $x^3 \le y \le 8$ and $0 \le x \le 2$ is revolved around the line $x = -2$. Use the shell method to find the volume (find an integral for it).

11) [8 pts] Begin the partial fractions method for solving this integral. You should go as far as writing out some fractions with $A$, $B$, etc in them, but you do not have to solve for the $A$, $B$, etc and do not have to compute any antiderivatives.
$\int \frac{x+3}{(x+1)^2(x^2+4)}\ dx$

12) [10 pts] Choose ONE proof to do, and circle it. If you use the back of the page, leave a note here.

A. Prove formula a, b or c from the table on page 1 of this exam using standard Ch.8 methods. Do not simply take a derivative of the right side.

B. State and explain in detail the Ch. 6.4 formula for surface area. Include a picture or two, a Riemann sum and several sentences. You can use a formula from 6.3, related to arc length, without proving it.

**Remarks + Scale:** The average was approx 62, with two high scores of 92. There was only one score between 51 and 71. The results were a bit better on the first 4 problems (approx 78%) and worse on 8 and 9 (approx 43%). An advisory scale for the exam, and also for your current semester average:

A's 70 - 100
B's 60 - 69
C's 50 - 59
D's 40 - 49

**Answers:**

1) $\pi/2$ using the trig identity which has appeared several times this term.

2) Set $u = \sec x$ and get $\int u^2\, du = \cdots = \frac{\sec^3 x}{3} + C$.

3) From part b of the table with $n = 2$, get $\frac{\tan^3 x}{3} - \int \tan^2 x\, dx$. To finish, do it again with $n = 2$, or use $-\int \tan^2 x\, dx = -\int \sec^2 x - 1\, dx = -\tan x + x + C$.

4) From part c of the table with $a = 2$, get $2\sin^{-1}(\frac{x}{2}) - \frac{x}{2}\sqrt{4-x^2} + C$. It is also OK, but much longer, to set $x = 2\sin\theta$.

5) $x \ln x - x + C$ from IBP. It is probably worthwhile to memorize this answer, but it is not very hard to derive.

6) $\int \frac{(x+1)+2}{x+1}\, dx = x + 2\ln|x+1| + C$. You could start with long division or with $u = x+1$ instead.

7) It is better to use the hint $x = 2y^{3/2}$ and $x' = 3y^{1/2}$, as in the text. We get $\int_0^1 \sqrt{1+9y}\, dy = \frac{1}{9}\int_1^{10} \sqrt{u}\, dy = \frac{2}{27}[10^{3/2} - 1]$.

Using $y = (x/2)^{2/3}$ is a bit dubious (see the text or lecture notes for the complaint) but it does lead to the correct answer in this example, and in that case I gave full credit.

8) From $f(x) = \sqrt{4-x^2}$ get SA $= \int_{-2}^{2} 2\pi\sqrt{4-x^2}\sqrt{1+\frac{x^2}{4-x^2}}\, dx = \cdots = 16\pi$.

9) $V = \int_0^7 62\pi(y/2)^2(10-y)\, dy$.

10) The radius is $x - (-2)$. The height is $8 - x^3$. So $V = \int_0^2 2\pi(x+2)(8-x^3)\, dx$.

11) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+4}$.

12) See the textbook for B. For (a), $1 - \cos ax = 2\sin(ax/2)$ and $\frac{1}{\sin} = \csc$, etc. For (c),

let $x = a \sin\theta$, etc. The best proof of (b) is not so similar to the other reduction formulas as I expected. But the proof is easy, if you start this way

$$\int \tan^n x \; dx = \int (\sec^2 - 1) \tan^{n-2} x \; dx = \int u^{n-2} \; du - \int \tan^{n-2} x \; dx$$

etc