Exam 2 | Mar 5, 2020
MAC 2312 | S Hudson

Name

Print your name above. Bring any electronics, papers, backpacks, etc to the front. Use the bathroom before the exam, if needed. Show all your work and reasoning in the space provided, or leave a note there. If you need more paper ask me and return it with your exam.

1) [10pts] Express the length of the curve $y = \frac{1}{2}(e^x + e^{-x})$, $0 \le x \le 1$, as a definite integral in standard form. For a little extra credit, evaluate the integral.

2) [10pts] A spring has a natural length of 10 in. An 800 lb. force stretches it to 14 in. How much work is required to stretch it from 10 in. to 12 in. ?

3) [10pts] Find the volume of the solid generated when the region between the graphs of $f(x) = 1/2 + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved around the x-axis.

4) [7 pts] Compute $\int x^2 \cos(x)\ dx$.

5) [7 pts] Compute $\int \sin(3x)\cos(4x)\ dx$.

6) [7 pts] Compute $\int \sin^3 x \cos^4 x\ dx$.

7) [7 pts] Compute $\int \frac{x}{x^2+2x+1}\ dx$

8) [7 pts] Compute $\int_1^4 \frac{(\ln x)^3}{2x}\ dx$.

9a) [5 pts] Compute $\int \sqrt{25 - x^2}\ dx$.

9b) [5 pts] Use 9a and a definite integral to compute the area inside the circle $x^2 + y^2 = 25$. Don't use πr^2 except maybe to check your answer.

10) [15 pts] Answer each with True or False. You do not have to explain.

There exist constants A and B such that $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{B}{x^2+1}$ for all x.

$\int \sec x\ dx = \ln|\sec x + \tan x| + C$

$\sin 5x \sin 6x = \frac{1}{2}[\cos x + \cos 11x]$ for all x.

$T_{100} > S_{100}$ (the Trapezoid and Simpson results) for the integral $\int_0^1 4x^3 + 5x\ dx$.

The volume of a sphere can be computed using either the Disk method or the Shell method.

11) [10 pts] Choose ONE proof. If possible, use sentences or formulas with complete justifications. Recall that the grading will be based on the clarity of your logic and explanations, as much as on any calculations involved.

A) Prove that $\ln(ab) = \ln(a) + \ln(b)$ using the definition of $\ln x$ (from Ch. 7.1). Justify any steps that rely on a definition, a theorem, or a technique of integration.

B) State and justify either the Shell Method formula (Ch 6.3) or the Arc Length formula (Ch 6.4). Either way draw a picture, then write out and explain a Riemann sum, etc, leading the standard integral formula. You have some freedom here, to use a dx or a dy integral, etc, but your formulas should match your picture, and you should not use any specific examples.

BONUS: [5 pts] Use a picture to compute $\int_0^1 \sin^{-1}(x)\, dx$.

Remarks, Scales, Answers: The average on the exam was approx 58% based on the top 23, with high scores of 98 and 80.

Most problems were taken from the textbook exercises, sometimes with small changes. See 6.3.14, 6.5.4, 7.1.11, 8.4.7, 8.5.17 (and basic ones from 8.2 and 8.3). The average on most problems was approx 50% to 60%, but a little higher on problems 1, 4 and 8 (approx 70%), and much worse on 9 (approx 20%). The advisory scale for Exam II is

A's 65 to 100
B's 55 to 64
C's 45 to 54
D's 35 to 44

To estimate your current semester grade, I averaged your 3 quiz scores (with total weight = 10) and your 2 exam scores (with total weight = 30) and wrote the result on your exam in the upper right. You should check this number. I have not included your HW yet, and may not have included all special cases (excuses, forgiveness, replacements, grade changes, effects from a possible Quiz3, etc). The average of these averages is approx 57%. Since this is similar to the Exam II average, you can use the scale above for your semester average too. For example, if the number in the upper right of your exam is a 53, you have a C+ average (not counting HW yet). If it is below 35, you have an F and should probably drop, or see me. Now the answers:

1) $\int_0^1 \sqrt{1 + ((e^x - e^{-x})/2)^2}\, dx = \int_0^1 e^{2x} + e^{-2x}\, dx$. You should simplify the first integral with algebra (though I gave lots of partial credit if you stopped there). The final answer for extra credit was $(e - 1/e)/2$.

2) $W = \int_0^2 200x\, dx = \cdots 400$ in-lbs. Recall that for Hooke's Law ($F = kx$) we use $x =$ the displacement from resting position. So you should not use numbers like 12 or 14 for x, not in the integral and not when finding $k = 800/4 = 200$.

3) The results weren't very good because most people did not sketch a good picture to find a decent plan. This is more important in problem 3 than problems 1 and 2 because there are good and bad options now. The parabola f lies above the line g (you can easily check that $f(1) > g(1)$, etc) and the picture shows that tall vertical rectangles make more sense

than wide ones. So, use washers with "dx". Most people did not draw any rectangles at all and did not reach this fairly simple conclusion.

The rest is a standard washer example, $V = \pi \int_0^2 (x^2 + 1/2)^2 - x^2 \; dx = \cdots = 69\pi/10$.

4) Apply IBP twice to get $x^2 \sin x + 2x \cos x - 2 \sin x + C$.

5) The trig identity for $\sin(a)\cos(b)$ (see Ch.8.3) leads to $\frac{1}{2} \int \sin(-x) + \sin(7x) \; dx = \frac{1}{2}(\cos(x) - \cos(7x)/7) + C$.

The main alternative is IBP twice, followed by "the trick", but that is harder. One student got this right, with the answer $\frac{3}{7}\cos(3x)\cos(4x) + \frac{4}{7}\sin(3x)\sin(4x) + C$.

6) Set $u = \cos x$ as usual in Ch. 8.3 and get $\frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$.

7) Since f has the form P/Q, use P.Fracs. Factor Q(x) and set $P/Q = \frac{A}{x+1} + \frac{B}{(x+1)^2}$ as instructed in Ch 8.5. With an incorrect formula here, you cannot solve for A and B, etc, in the next step. Get $\int \frac{1}{x+1} - \frac{1}{(x+1)^2} = \ln|x+1| + \frac{1}{x+1} + C$.

8) From $u = \ln x$, and the simplification $\ln 1 = 0$, get $\frac{(\ln 4)^4}{8} + C$ and stop. Though $\ln(4^4) = 4 \ln 4$, this doesn't simplify the answer further. The parentheses matter.

9a) This is 8.4.7. Set $x = 5 \sin\theta$ and get $25 \int \sqrt{\cos^2\theta} \cos\theta \; d\theta = 25 \int \frac{1+\cos 2\theta}{2} \; d\theta$ (using the trig identity from Day 1, etc). So, $\frac{25}{2}(\theta + \frac{\sin 2\theta}{2}) = \frac{25}{2}(\sin^{-1}\frac{x}{5} + \frac{x}{5}\sqrt{1 - (\frac{x}{5})^2})$.

The final simplification is somewhat optional but it helps with 9b. Not many people got 9a completely right, so the results on 9b were low.

9b) From the previous answer and the FTC, the area in quadrant 1 is $F|_0^5 = \frac{25}{2}\sin^{-1} 1 = \frac{25\pi}{4}$. We can multiply that by 4 to get the area of the whole circle, which agrees with the usual formula, πr^2 with $r = 5$.

10) FTFTT

11) These are in the text and/or the lecture notes. Some answers did not make much sense (maybe they were semi-memorized) and most did not go into enough depth. For the Shell Method, for example, a good justification should include a picture of a shell and a discussion of its volume. This would lead to a Riemann sum and an integral. For the $\ln ab$ option, you should ideally mention the definition of $\ln x$ three times, and the abc theorem, and show the u-substitution.

Bonus) See me. In brief, the region is part of a rectangle with corners at $(0,0)$ and $(1, \pi/2)$. The other part of the rectangle is easier, with area $\int_0^{\pi/2} \sin(y) \; dy = 1$. So, subtract this from the area of the rectangle and get $A = \pi/2 - 1$.

You could also compute $\int \sin^{-1} \; dx$ using IBP, but that was not the intent here.