Exam III Nov 7, 2016
MAC 2312 S Hudson

1) [8 pts each] Solve each, or show it diverges. Use a limit with any improper integrals. Include $C$'s, as needed.

$\int \frac{dx}{x(x^2-4)}$

$\int \frac{dx}{x^2\sqrt{x^2-9}}$.

$\int_2^\infty \frac{dx}{x\ln(x)}$

2) [10 pts total] You can use the approximate data and remarks below 2a for either 2a or 2b. Both are about approximating $\int_0^3 \sqrt{x+3}\, dx$.

2a) Use the Trapezoid Rule with $n = 3$ to approximate this integral. Simplify your answer using decimal notation (give an answer such as 3.81).

Data:
$\sqrt{2} = 1.41$
$\sqrt{3} = 1.73$
$\sqrt{5} = 2.24$
$\sqrt{6} = 2.45$
$\sqrt{7} = 2.65$

Remark: If $f(x) = \sqrt{x+3}$ then $f''(0) = -1/20$ and $f''(3) = -1/60$.

2b) Recall that $|E_T| \leq \frac{(b-a)^3 K_2}{12n^2}$. How large must $n$ be so that the error when using $T_n$ is at most 1/8000 ?

3) [8 pts] This is exercise 47c from Ch.9.1, about the Fibonacci sequence $1, 1, 2, 3, 5, 8\ldots$. You can use the formula from 47a, that $\frac{a_{n+2}}{a_{n+1}} = 1 + \frac{a_n}{a_{n+1}}$ and can assume that $\lim \frac{a_{n+1}}{a_n} = L$ exists. Compute $L$.

4) [8 pts] Use a geometric series to express the repeating $0.4444\ldots$ as a fraction of two integers.

5) (5 pts each) State whether each sum converges or diverges, and justify (usually by naming a convergence test and showing some work).

$\sum_{k=1}^\infty \frac{\sqrt{k}}{k^2+3}$

$\sum_{k=1}^\infty \frac{4^k}{k^2}$

$\sum_{k=1}^\infty (\frac{k}{100})^k$

6) (7 pts) Classify as D, CC or CA, and justify: $\sum_{k=3}^\infty \frac{(-1)^k}{k\ln(k)}$.

7) (20 pts) Answer True or False:

$\lim_{n\to\infty} \cos(2n\pi)$ diverges.

Every increasing sequence that is bounded above converges.

The alternating harmonic series converges conditionally.

The alternating harmonic series can be rearranged to converge to 0.

Simpson's approximation $S_{100}$ is a weighted average of $M_{50}$ and $T_{50}$.

Simpson's approximation $S_{100}$ for $\int_1^7 x^4 + x^3 + x^2 + 4\ dx$ is exact.

The midpoint approximation $M_n$ is the average of $R_n$ and $L_n$.

The Divergence Test is inconclusive for the series $\sum_{k=1}^{\infty} \frac{k!}{13^k}$

The Ratio Test is inconclusive for the series $\sum_{k=3}^{\infty} \frac{k}{k^7+13}$

The partial fraction decomposition for $\frac{(x+2)^3}{(x+1)^3(x^2+1)}$ has exactly 4 unknowns (A,B,C,D).

8) (8 pts) Choose ONE proof, explain thoroughly:

a) State and prove the Comparison Test. You can omit part b, the contrapositive part.

b) State the Theorem 9.3.3 about convergence of a geometric series and prove it. For full credit prove the formula for $s_n$ too.

BONUS: (5 pts) Decide whether the sum converges or diverges, and *prove* your answer using the Ratio Test. Partial credit is not likely.

$$\sum_{k=3}^{\infty} \frac{17k!}{k^k}$$

**Remarks, Answers:** The average was 56 out of 100, with highs of 92 and 87. The results on most of the problems were similar except for problems 3 (11%) and 4 (28%). Problem 3 was an assigned HW problem and 4 was similar to one. Here is the scale for the exam. You can use the same scale for the average of your three exams so far.

A's 65 to 100
B's 55 to 64
C's 45 to 54
D's 35 to 44

1a) $\frac{-1}{4}\ln|x| + \frac{1}{8}\ln|x+2| + \frac{1}{8}\ln|x-2| + C$ using partial fractions.

1b) $\frac{\sqrt{x^2-9}}{9x} + C$

1c) Diverges. Use a limit and $u = \ln(x)$.

2a) 6.33

2b) $n = \sqrt{900} = 30$. A common mistake was setting $K_2 = 1/60$ or even $-1/60$. It should be the larger number, $1/20$. But arithmetic errors were also very common.

3) Use $L = 1 + 1/L$ to get $L = \frac{1+\sqrt{5}}{2}$.

4) $a/(1-r) = 4/9$. I used $a = 4/10$, $r = 1/10$ but a few people used $a = 44/100$, $r = 1/100$ and got $44/99$ which is the same.

5) Brief answers only - more work should be included. Other explanations are possible, though I don't recall many successful alternatives.

- a) Converges, by comparison with a p-series, $p = 3/2$.
- b) Diverges, Ratio T.
- c) Diverges, Root Test.

6) CC, using the AST and the integral test. Other tests are not likely to work well on this one.

7) FTTTT FFFTF

8) See the text. The results were pretty low for part a; few people even mentioned partial sums. The results were better for part b, but with many minor errors (confusing $s_n$ with $S$, not stating the theorem at the start, not mentioning the case $|r| \geq 1$, etc).

B) $\rho = \cdots \lim \frac{k^k}{(k+1)^k} = \cdots 1/e < 1$, so the series converges. I don't think there is any good way to do this without $e$ appearing.