Exam III April 6, 2017
MAC 2312 S Hudson

1) [8 pts each] Solve each, or show it diverges. Use a limit with any improper integrals. Include the $+C$, if needed.

$\int \frac{dx}{x(x^2+4)}$

$\int \frac{x^3}{x^2-1}\, dx$

$\int_2^\infty \frac{dx}{x(\ln(x))^2}$

2) [10 pts total] You can use the approximate data and remarks below 2a for either 2a or 2b. Both are about approximating $\int_0^3 \sqrt{x+3}\, dx$.

2a) Use the Trapezoid Rule with $n = 3$ to approximate this integral. Simplify your answer completely using decimal notation (give an answer such as 3.81).

Data:
$\sqrt{2} = 1.41$
$\sqrt{3} = 1.73$
$\sqrt{5} = 2.24$
$\sqrt{6} = 2.45$
$\sqrt{7} = 2.65$

Remark: If $f(x) = \sqrt{x+3}$ then $f''(0) = -1/20$ and $f''(3) = -1/60$.

2b) Recall that $|E_T| \leq \frac{(b-a)^3 K_2}{12n^2}$. How large must n be so that the error when using T_n is at most 1/8000 ?

3) [5 pts] Compute $\lim \cos((n + \frac{1}{2})\pi)$.

4) [8 pts] Use a geometric series to express the repeating $0.464646\ldots$ as a fraction of two integers.

5) (5 pts each) State whether each sum converges or diverges, and justify (usually by naming a convergence test and showing some work).

a) $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2+7}$

b) $\sum_{k=1}^{\infty} \frac{(-3)^{k+1}}{9^{k-1}}$

c) $\sum_{k=1}^{\infty} (\frac{2k}{k+2017})^k$

d) $\sum_{k=1}^{\infty} ((k+2)^3) - k^3$

e) $\sum_{k=3}^{\infty} \frac{(-1)^k}{k\ln(k)}$.

6) (20 pts) Answer True or False:

Simpson's Rule on $\int_0^6 x^4\ dx$ with $n = 10$ produces an exact answer (so $E_S = 0$).

The Midpoint Rule on $\int_0^7 x^4\ dx$ with $n = 10$ gives an overestimate, $M_{10} \geq \int_0^7 x^4\ dx$.

The alternating harmonic series converges conditionally.

The alternating harmonic series can be rearranged to converge to 2017.

The Divergence Test is inconclusive for the series $\sum_{k=1}^{\infty} \frac{13^k}{k}$

The Ratio Test is inconclusive for the series $\sum_{k=3}^{\infty} \frac{20k}{k+17}$

If $\sum a_k$ converges then $\lim a_k = 0$.

$\int_1^{\infty} x^p\ dx$ converges if and only if $p < -1$.

If a sequence is eventually decreasing, then it is bounded above.

If $\sum a_k$ converges, then its partial sums are monotonic.

7) (8 pts) Choose ONE proof, explain thoroughly:

a) State and prove the Comparison Test. You can omit part b, the contrapositive part.

b) State the Theorem 9.3.3 about convergence of a geometric series and prove it. For full credit prove the formula for s_n too.

Bonus [about 5 points]: Compute $\sum_{k=1}^{\infty} \frac{7}{k^2+3k}$ and simplify.

Remarks and Answers: The average among the top 20 was approx 63, with high scores of 99 and 83. The results were similar on most of the problems, except problem 4 (80%). The advisory scale for the exam is

A's to 71 to 100
B's to 61 to 70
C's to 51 to 60
D's to 41 to 50

For your semester standing, average your three exam scores and use the scale below:

A's to 69 to 100
B's to 59 to 68
C's to 49 to 58
D's to 39 to 48

1a) From PF's, and then $u = x^2 + 4$, get $\frac{1}{4}\int \frac{1}{x} - \frac{x}{x^2+4}\ dx = \frac{1}{4}\ln|x| - \frac{1}{8}\ln|x^2+4| + C$. With an indefinite integral like this, you always include the +C and you never answer "diverges".

1b) After dividing, get $\int x + \frac{x}{x^2-1}\ dx = \frac{x^2}{2} + \frac{\ln|x^2-1|}{2} + C$. You can use PF's on $\int \frac{x}{x^2-1}\ dx$

(though that is slower than using u), but not on the original integral, because deg P > deg Q.

1c) $\frac{1}{\ln 2}$. Use a limit as usual and use $u = \ln(x)$. Do not include +C since this is a definite integral.

2a) 6.33

2b) 30. I also accepted similar answers such as $n \geq 30$ or $n = 31$. The hint/remark leads to $K_2 = 1/20$.

3) 0. Check that $a_0 = \cos(\pi/2) = 0$, and $a_1 = 0$, etc.

4) 46/99.

5) In general, I gave at least 3 points for answering C or D correctly (with some plausible attempt at justification). No partial credit without that. Answer to 5a): C, using the Comparison Test (or LCT) with the p-series $\sum 1/k^{3/2}$, with enough work shown.

5b) C. This is geometric with $r = -1/3$, so that $|r| < 1$. You can use RTAC if you prefer and get $\rho = 1/3 < 1$. The AST is OK, but requires perhaps a bit more work for a complete answer.

5c) D. The Root Test leads to $\rho = 2 > 1$.

5d) D, by the Div.Test (check that $\lim a_k \neq 0$. Or, use the telescoping method (compute s_n explicitly). Also the Comp.T, etc, are OK.

5e) C. The only simple method I see is the fairly obvious AST.

6) FFTTF TTTTF

7) The results were OK on 7b), but most people chose 7a), and did not seem to have studied that proof. Both proofs involve studying partial sums. For 7b), the main thing is to explain that the partial sums of $\sum b_k$ will be bounded but bigger than those of $\sum a_k$.

B) Use PF's to get $\frac{7}{3} \sum \frac{1}{k} - \frac{1}{k+3}$ (several people got this far but no further) which is *telescoping*. Get $\frac{7}{3}(1 + 1/2 + 1/3) = 77/18$.