Exam III Nov 12, 2020

MAC 2312 S Hudson

1) [8 pts each] Solve each, or show it diverges. Use a limit with any improper integrals.

$\int \frac{dx}{x(x^2+4)}$

$\int_2^\infty \frac{dx}{x(\ln(x))^2}$

2) [8 pts] Use a geometric series to express the repeating $0.2020\overline{20}\ldots$ as a fraction of two integers.

3) [10 pts total] 3a) Approximate $\int_0^3 \sqrt{x+3}\ dx$ using the Trapezoid Rule with $n = 3$. Simplify your answer completely using decimal notation (give an answer such as 3.81). You can use the following approximate data.

$\sqrt{2} = 1.41$
$\sqrt{3} = 1.73$
$\sqrt{5} = 2.24$
$\sqrt{6} = 2.45$
$\sqrt{7} = 2.65$

Remark: If $f(x) = \sqrt{x+3}$ then $f''(0) = -1/20$ and $f''(3) = -1/60$.

3b) Recall that $|E_T| \leq \frac{M(b-a)^3}{12n^2}$ where M is related to f''. How large must n be so that the error when using T_n is at most $1/8000$?

4) (6 pts each) State whether each sum converges or diverges, and justify, usually by naming a convergence test and showing some work.

a) $\sum_{k=1}^{\infty} \frac{5\sqrt{k}}{k^2+7}$

b) $\sum_{k=1}^{\infty} \frac{e^n}{5+e^n}$

c) $\sum_{k=1}^{\infty} (\frac{2k}{k+2021})^k$

d) $\sum_{k=1}^{\infty} \frac{\ln k}{k}$. Typo corrected on 11/16.

e) $\sum_{k=3}^{\infty} \frac{(-1)^k}{k\ln(k)}$.

5) (6 pts) Find the interval of convergence of $\sum_{k=0}^{\infty} \frac{x^k}{2^k k^2}$.

6) (20 pts) Answer True or False:

If $\sum(a_k - 2)$ converges then $\lim a_k = 2$.
$\int_0^1 x^{-3}\ dx$ converges.

If a sequence is decreasing, then it is bounded above.

If $\sum(a_k + b_k)$ converges then $\sum a_k$ and $\sum b_k$ both converge.

Simpson's Rule on $\int_0^6 x^3 + x^2 \; dx$ with $n = 10$ produces an exact answer.

If $\sum a_k$ converges, then its partial sums are bounded.

The harmonic series converges conditionally.

The alternating harmonic series can be rearranged to converge to 2017.

The Divergence Test is inconclusive for the series $\sum_{k=1}^{\infty} \frac{13^k}{12^k + k}$

The Ratio Test is inconclusive for the series $\sum_{k=3}^{\infty} \frac{k^2}{k^2+7}$

7) (10 pts) Choose ONE proof, explain thoroughly:

a) State and prove the Comparison Test using partial sums. You can omit the contrapositive part.

b) State the main theorem about convergence of a geometric series and prove it. For max credit, prove the formula for s_n too.

Bonus [about 5 points]: Compute $\sum_{k=1}^{\infty} \frac{5}{k^2+5k}$.

Remarks, Answers: The average was again approx 60% so this exam has the same scale as Exam 2. The high scores were 86 and 84. The average scores were pretty similar on all the problems, except for # 5 (28%).

1a) Using partial fractions, $\frac{1}{4} \ln |x| - \frac{1}{8} \ln |x^2 + 1| + C$.

1b) Converges to $\frac{1}{\ln 2}$. Use a $\lim_{M \to \infty}$ as usual, and set $u = \ln x$, etc.

2) $\frac{20}{99}$. Use $S = \frac{a}{1-r}$ with $a = 0.20$ and $r = 0.01$.

3a) 6.33

3b) Set $M = 1/20$ (not $-1/20$) and get $n = \sqrt{900} = 30$. I would probably accept a similar number, such as 31, if the method were correct.

4a) C. Compare it to a p=series with p = 3/2.

4b) D. The easiest method is the Divergence Test, but a few people succeeded with the Integral Test. I don't recall grading any other successful methods. I think the Comparison Test is *always* an option for a positive series, but it can be tricky.

4c) D. The Root Test is Best. The Div Test is possible.

4d) D. I expected people to use the Integral Test. It is also easy to compare with the

Harmonic Series, since $\ln k > 1$ (eventually).

4e) C. Use the A.S.Test. I see no other options. This series does not "CA", so removing the $(-1)^k$ will not succeed. It might be worth a try if the AST were not so easy.

5) $I = [-2, 2]$. The results on this one were rather low, so here is an outline of the work. As usual, start with the RTAC:

Part A: $\rho = \lim \cdots = |x|/2$. Set $|x|/2 < 1$ and get $-2 < x < 2$. This might be I, but we don't know yet if $x = \pm 2$ give C.

Part B1: Set $x = 2$ and cancel the 2's. We get a p-series with p=2, which converges. So 2 gives C, $2 \in I$.

Part B2: Set $x = -2$ and get C again (using either AST or CA).

Answer: $I = [-2, 2]$.

6) TFTFT TFTFT

7) See the text or lecture notes.

Bonus) I was pleasantly surprised that so many people used partial fractions to make this into a telescoping series, $\sum(\frac{1}{k} - \frac{1}{k+5})$, but nobody quite got to the correct final answer. If you write out s_6 for example, you will see the pattern. The $1/k$ term cancels out with some $-1/(k+5)$ term if $k > 5$ (but not if k=1,2,3,4,5). The answer is $S = 1 \ + 1/2 \ + 1/3 \ + 1/4 \ + 1/5$, which you did not have to simplify.